

This Talk:



THE NATURE OF TURBULENCE IN SPACE PLASMAS

A Mission and Science Overview

Kristopher Klein, HelioSwarm Deputy Principal Investigator
University of Arizona

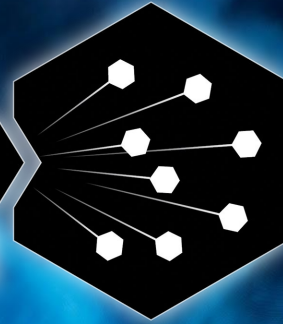
May 21, 2026

The heliospheric space plasma physics in the era of multipoint space missions





HELIO SWARM



THE NATURE OF TURBULENCE IN SPACE PLASMAS

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1 UNH, 2 LESIA/Paris, 3 UChicago, 4 Harvard/SAO, 5 NASA/GSFC, 6 Imperial College, 7 UCL/MSSL,

8 IRAP/CNRS, 9 UIowa, 10 NASA/ARC, 12 UArizona, 13 Princeton, 14 LPP/CNES, 15 UC Berkeley, 16 UDel,

17 Cambridge, 18 Aberystwyth University, 19 Oxford, 20 LANL, 21 UAH, 22 UWisc





Agenda

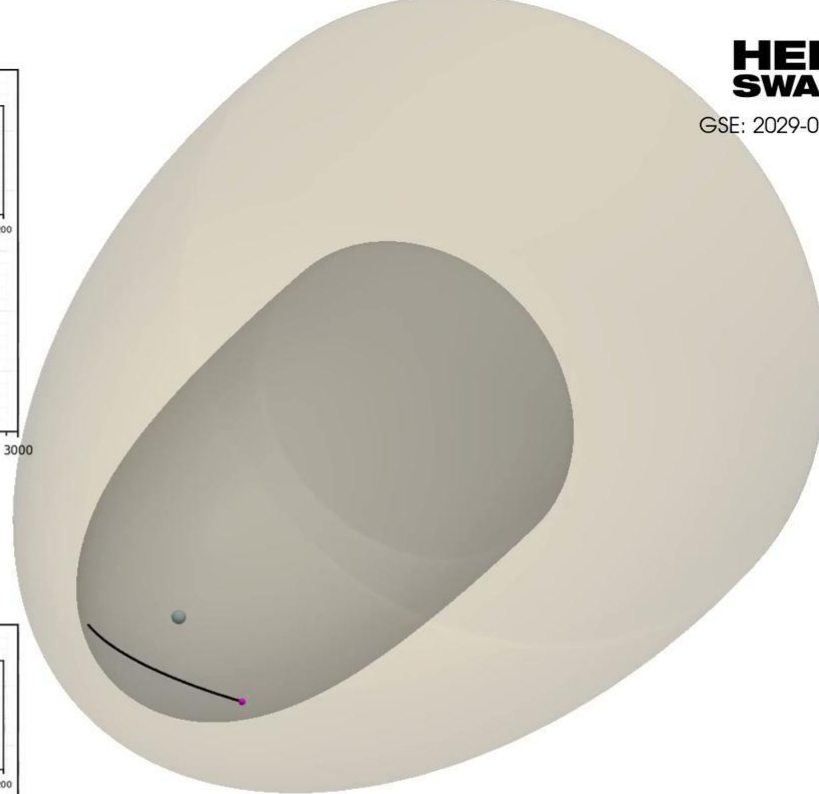
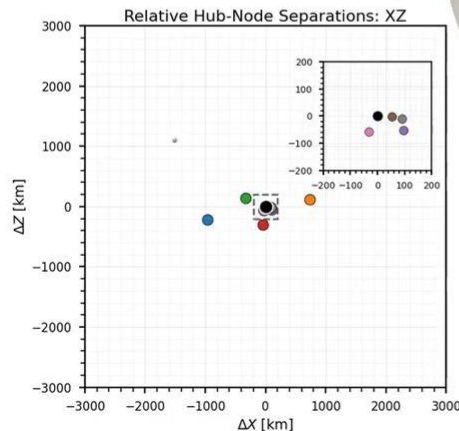
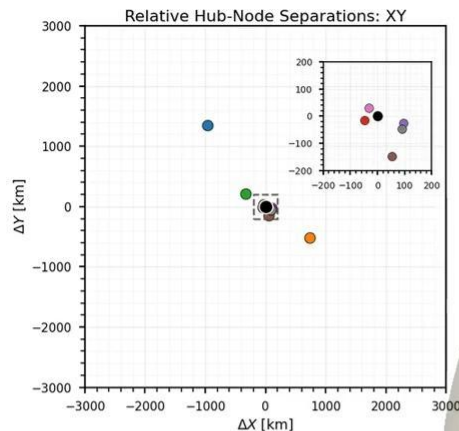
A (Short) Plasma
Turbulence Primer

The Need for
Multi-Scale
Observatories

A HelioSwarm
Overview



GSE: 2029-08-07 10:00 UTC



Drivers: 2020-01-01T04:00:00Z
P_{dyn}=1.06 nPa D_{st}=4.5 nT Ma=8.05
Shue 1998 Magnetopause + conic Bowshock.



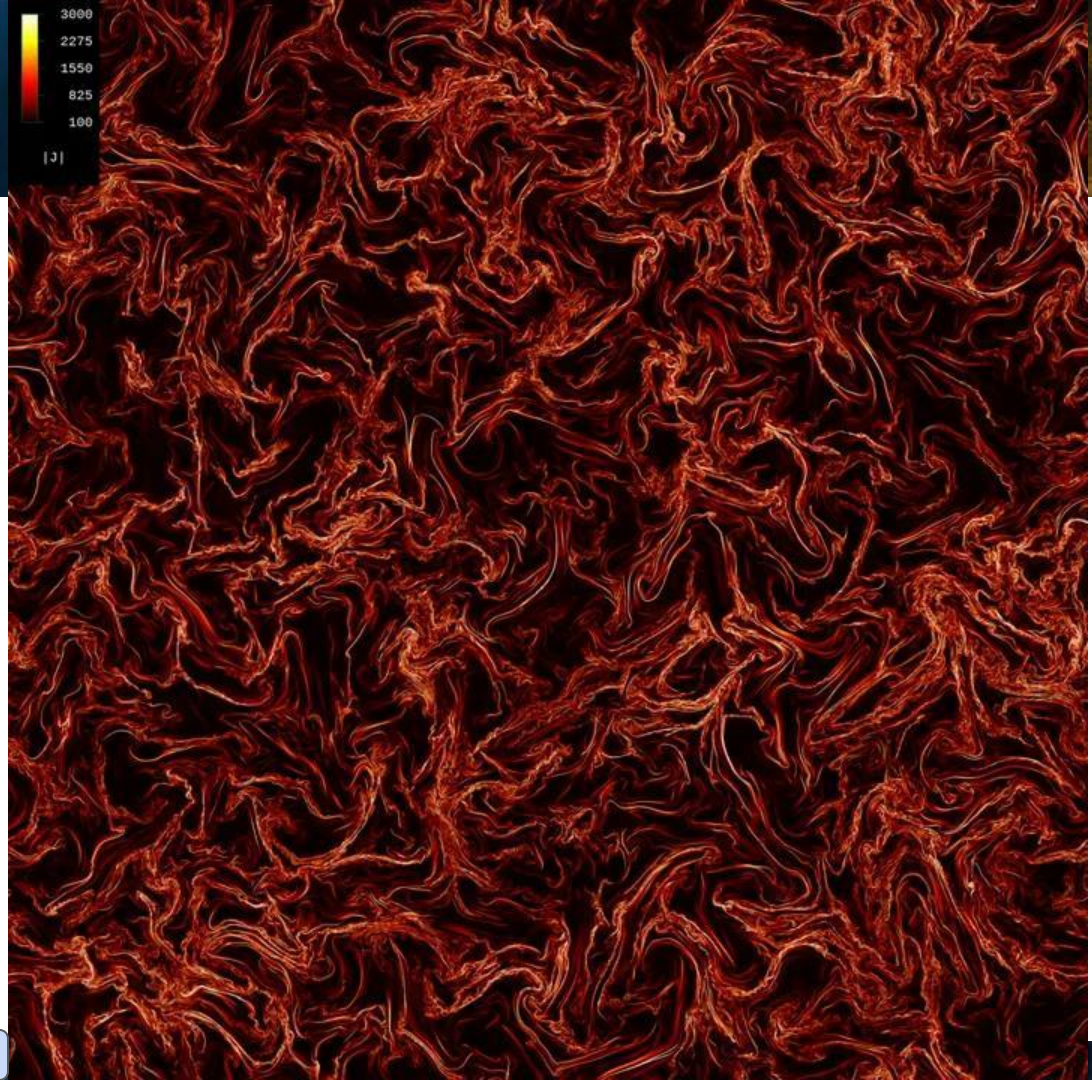
Turbulence is Multiscale Disorder

Turbulence is a statistical description of nonlinearly interacting fluctuations.

These waves or eddies shear each other apart, cascading energy from larger to smaller scales.

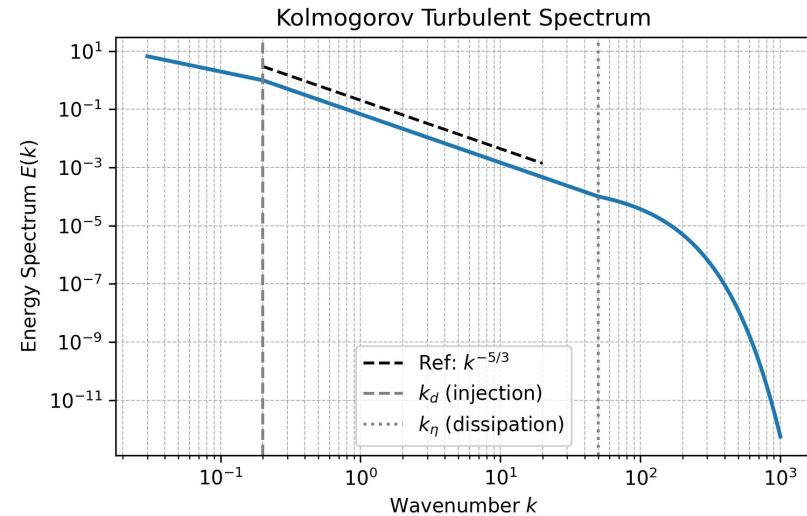
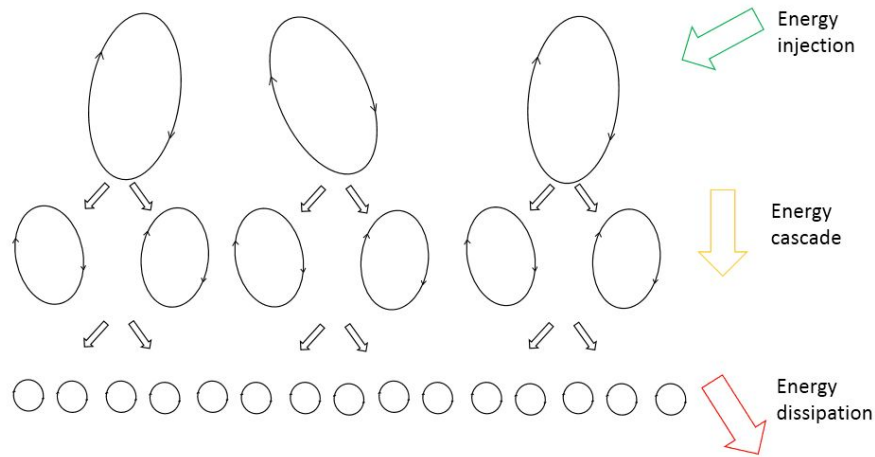
The energy concentrates in small regions of space, where the turbulence dissipates, heating the plasma.

“Big whirls have little whirls,
That feed on their velocity;
And little whirls have lesser whirls,
And so on to viscosity.”
— Lewis Fry Richardson





The **turbulent cascade** connects large-scale motions with ever smaller fluctuations, and eventual heating.



We want to characterize how energy, mass, and momentum is

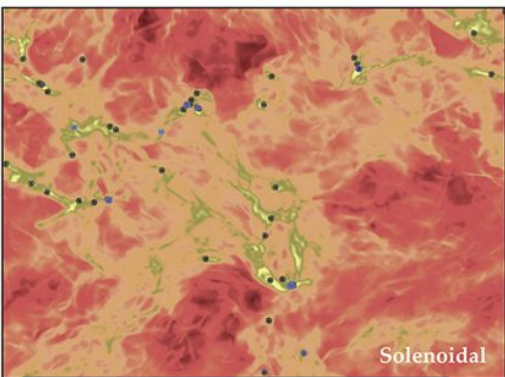
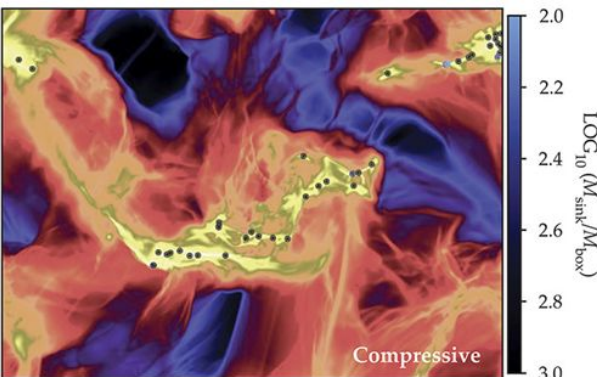
- distributed within,
- transported through, and
- and removed from

a turbulent medium, including dependencies fluid characteristics.



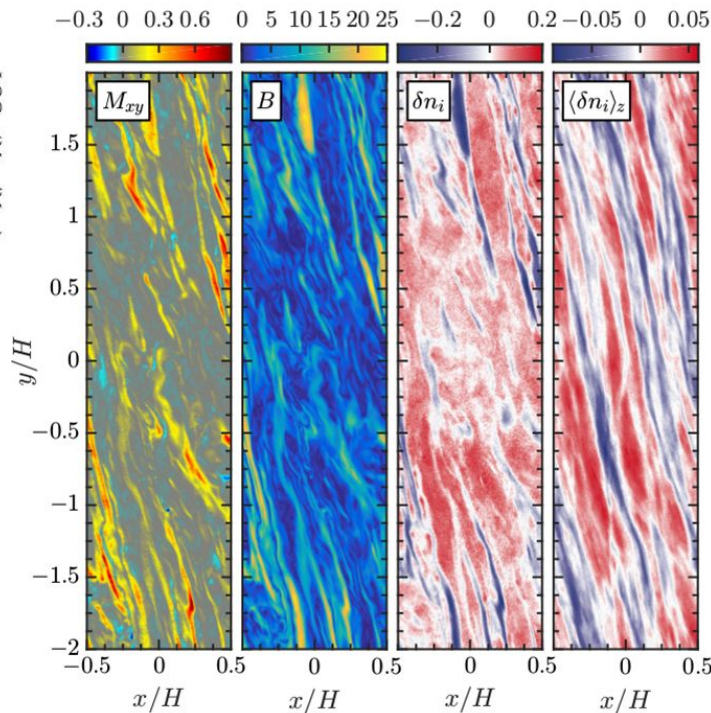
Turbulence Plays a Fundamental Role in Universal Plasmas

Seeds dense regions for star formation
(e.g. McKee & Ostriker 2007)



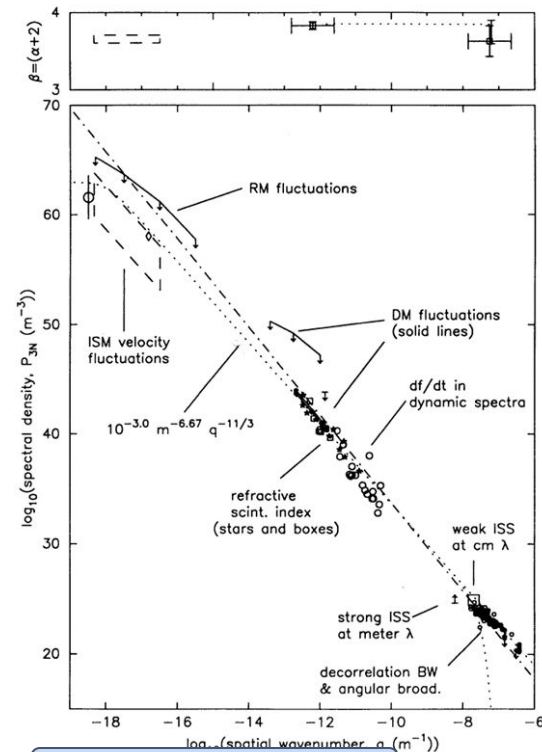
Federrath & Klessen 2012

Enables angular momentum transport in accretion disks
(e.g. Balbus & Hawley 1998)



Kunz et al 2016

Determines the dispersal and mixing of elements in the ISM (e.g. Scalo & Elmegreen 2004)



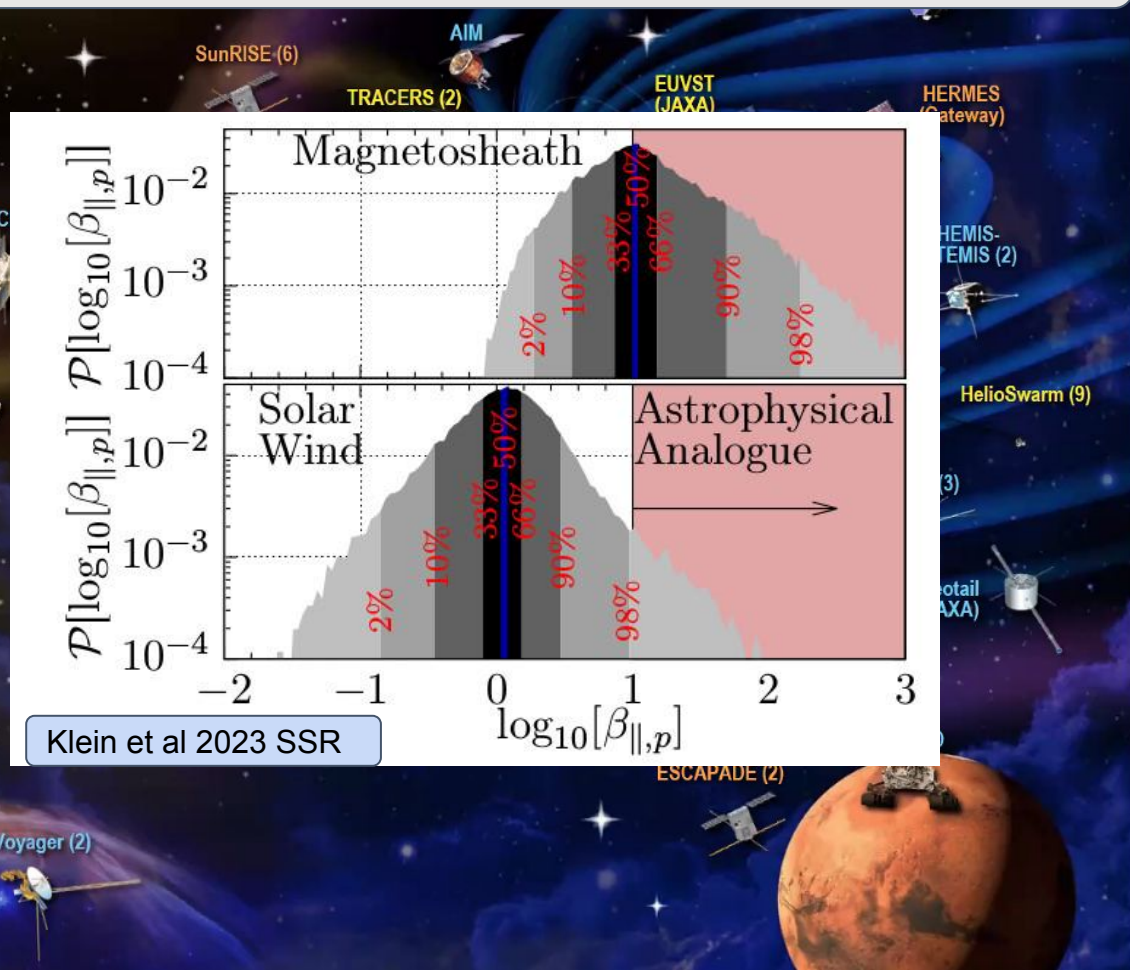
Armstrong et al 1995

The Solar Wind as a Natural Laboratory for Studying Plasma Turbulence

- 20 Operating Missions with 27 Spacecraft
- 14 Missions in Formulation or Implementation

We take advantage of simultaneous measurements of fields and particles to study fundamental plasma processes relevant to solar and astrophysical environments.

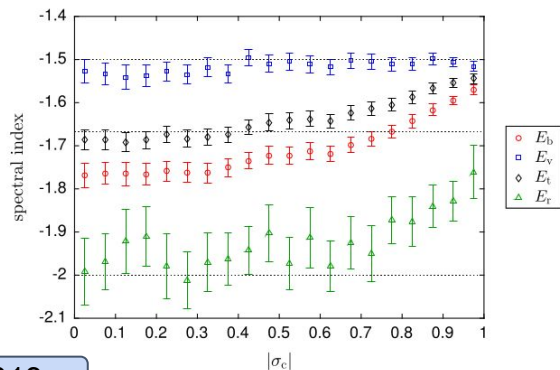
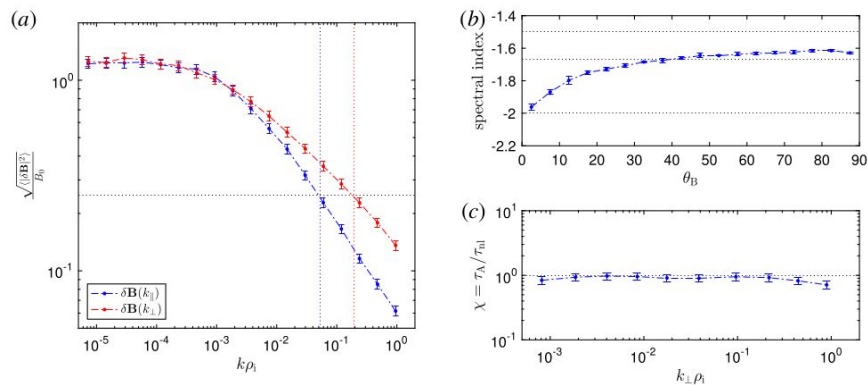
For statistical studies of plasma conditions at 1 au, see Klein & Vech RNAAS 2019, Wilson et al ApJS 2018, and “**A Quarter Century of Wind Spacecraft Discoveries**” by Wilson et al 2021.



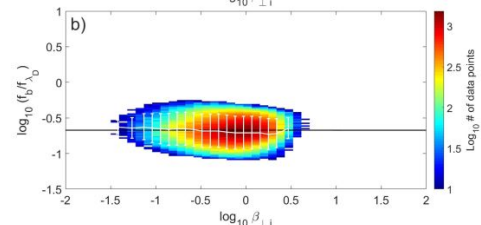
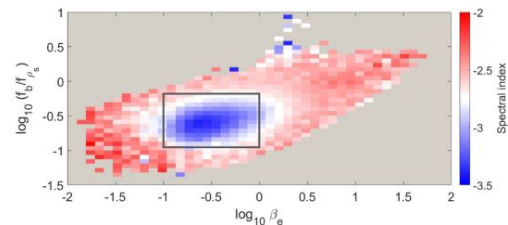
OPERATING & FUTURE



Progress in Plasma Turbulence from Solar Wind Observations



Chen et al 2013



Vech et al 2018 ApJL

Statistical analysis of single point observations from Wind, Parker Solar Probe, and Solar Orbiter— appropriately conditioned— have enabled testing of various turbulence theories, e.g. spectral slopes (Woodham et al 2018, Bowen et al 2024, Sioulas et al 2024), strength of nonlinearities (Mallet et al 2015), occurrence of magnetic reconnection interrupting turbulence (Mallet et al 2017, Loureiro & Boldyrev 2017), and correlation scales (Matthaeus et al 2005, Weygand et al 2011).

See the excellent review article [Chen 2016 JPP](#) for more details

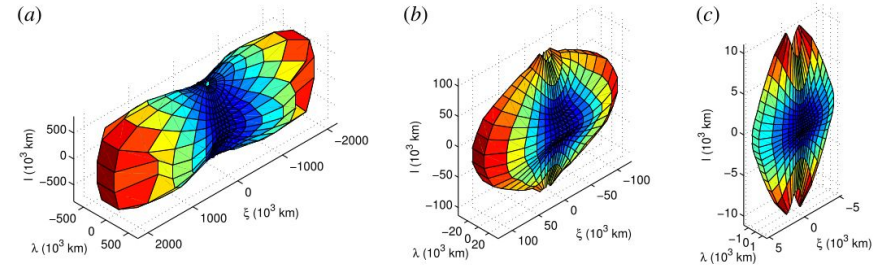


Unresolved Questions about Structure and Dynamics



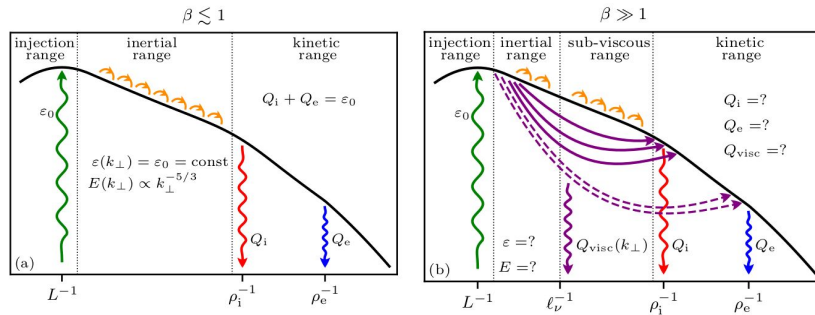
Despite claims we have reached “the end of the road” (Schekochihin 2022 JPP) in understanding (R)MHD turbulence, there are many outstanding questions about turbulent systems, especially as kinetic processes start to play a significant role.

How is Energy Distributed in Space and Time?

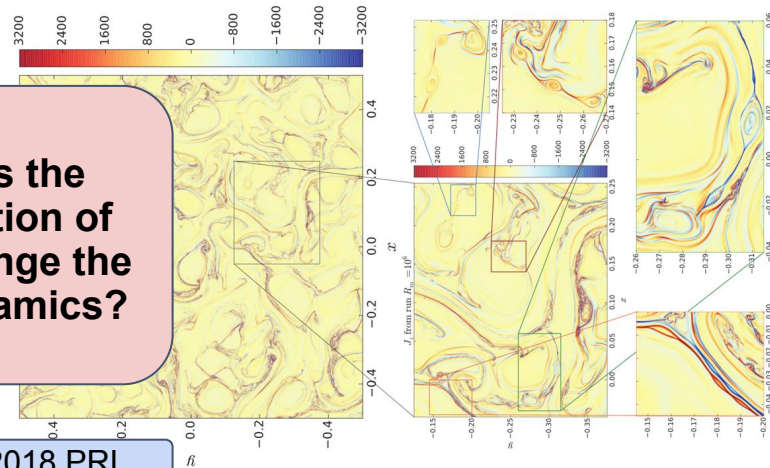


Chen et al 2012 ApJ

How is Energy Cascaded Between Scales?



How does the concentration of energy change the thermodynamics?



Dong et al 2018 PRL

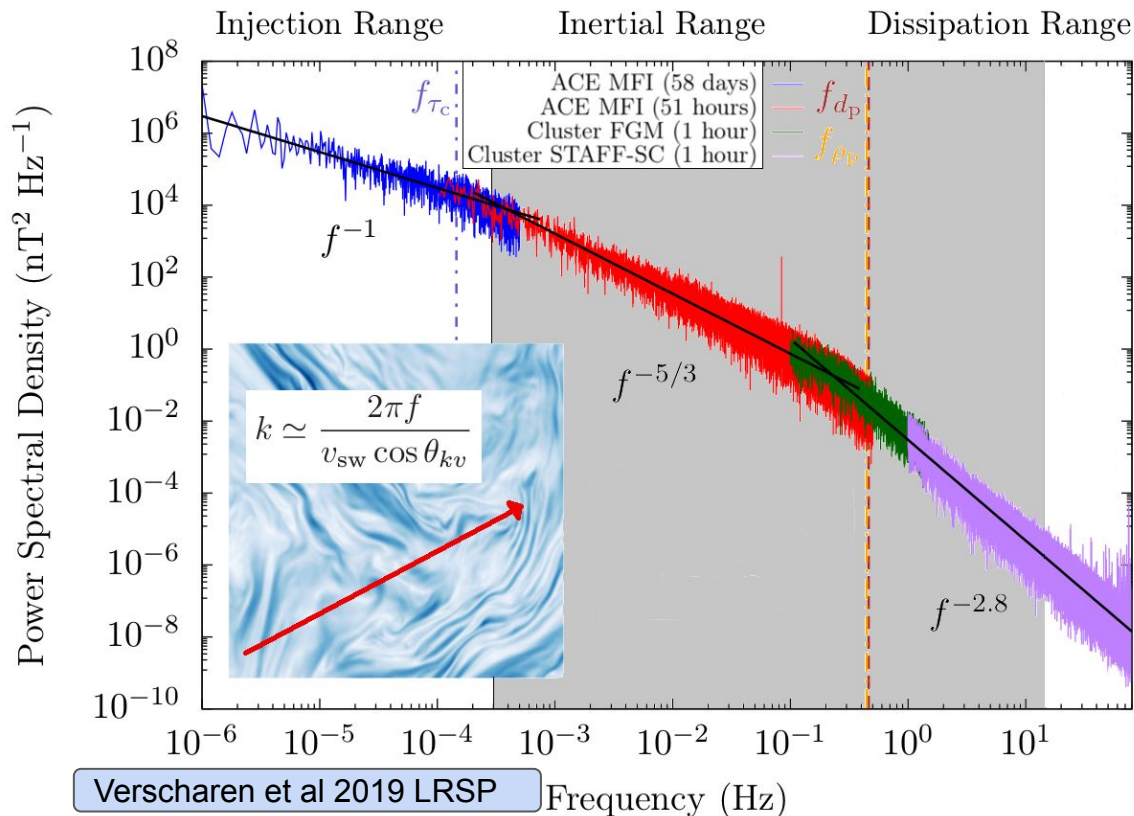
Arzamasskiy et al 2023 PRX



Turbulence is Multiscale Disorder, Frequently Studied at One Point



Single-spacecraft rely on simplifications to disentangle spatial and temporal dynamics and combine many different types of turbulence in order to have a statistically large enough data set.



Report of the NASA Plasma Turbulence Explorer Study Group



September 1980

National Aeronautics and
Space Administration

Jet Propulsion Laboratory
California Institute of Technology
Pasadena, California

Montgomery et al. 1980

Back in 1980, a study group called for launching N spacecraft into the near-Earth environment as a means of studying the nature of the turbulence in weakly collisional plasmas.

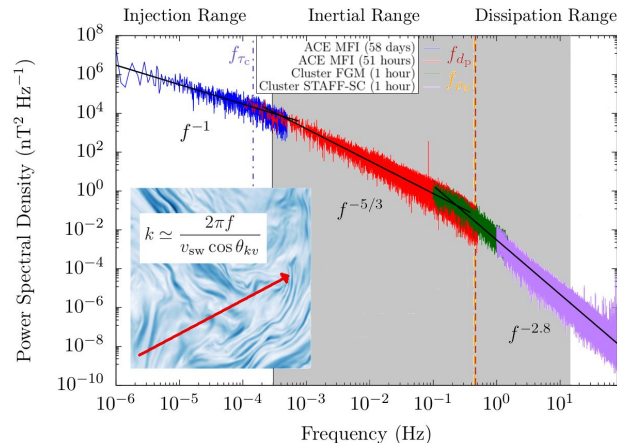
F. Number of Spacecraft and Their Configuration

A mission aimed at studying turbulence requires simultaneous measurements from N spacecraft, where N is at least 2. The optimum value of N very likely depends on the kind of problem being studied. The experience of meteorology is that even two stations can be extremely effective for separating spatial and temporal variations of certain phenomena, provided that the location of the stations is carefully selected. Generally speaking, $N = 3$ is far more powerful than $N = 2$, and $N = 4$ is even better, if the locations are judiciously selected. The point at which increasing N by one is not desirable is largely determined by economic considerations.

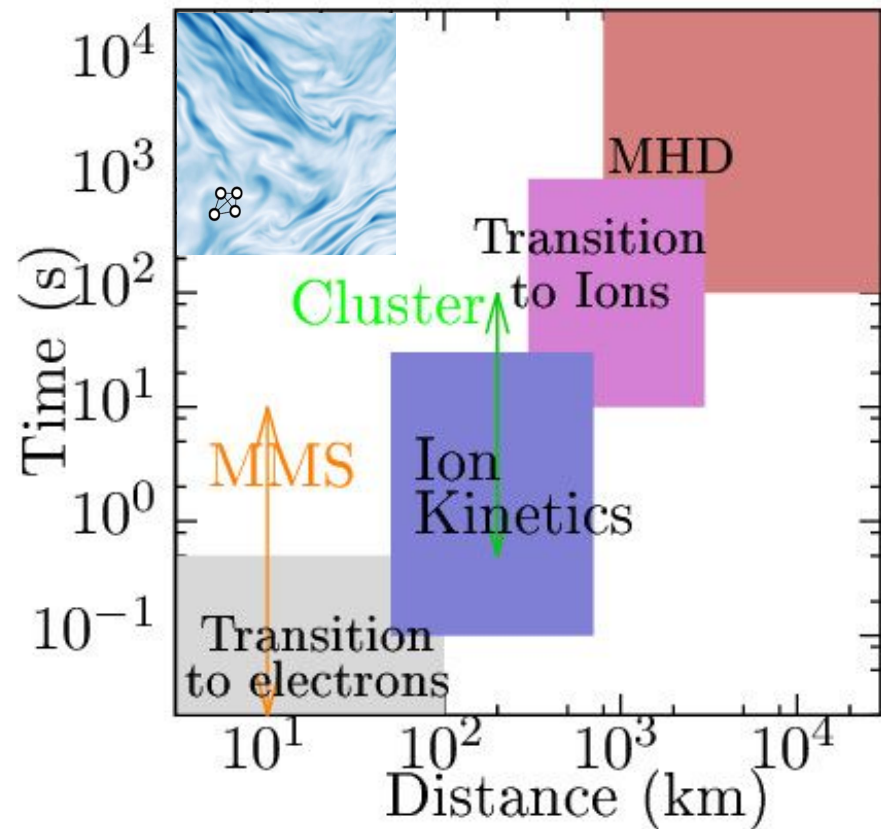
“Looking to the future I believe that progress requires bunches of satellites, though these are as yet in no published program....[S]ince satellites are being launched singly, the scientific returns are less than they could be.” - James Dungey [1966]



Turbulence is Three-Dimensional; A **Multi-Point** Observatory is Needed

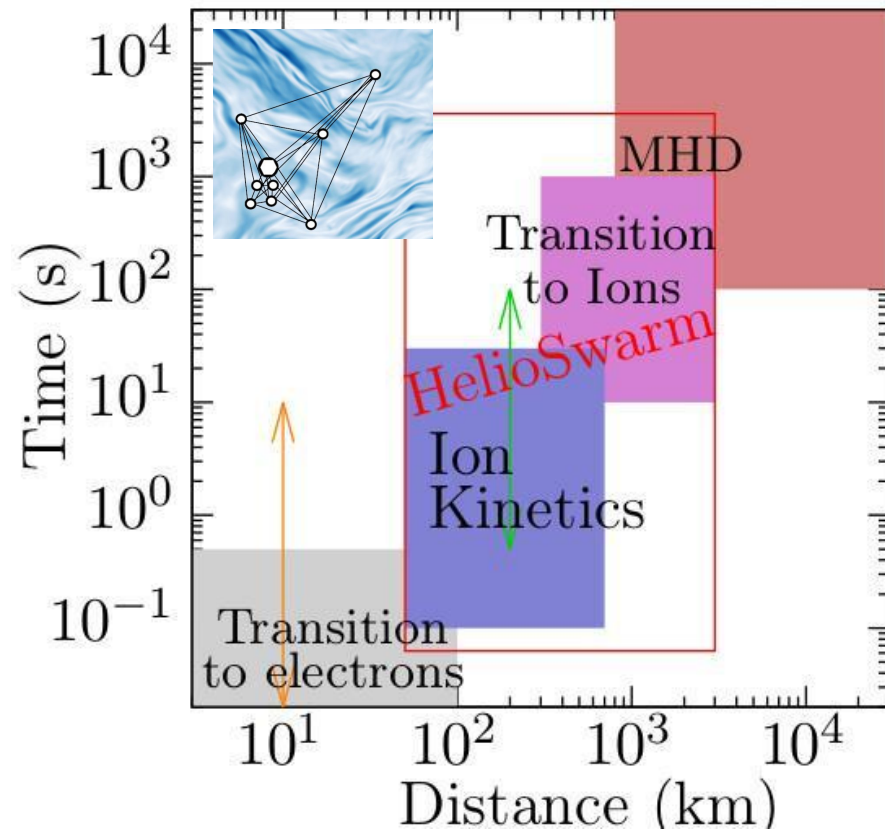
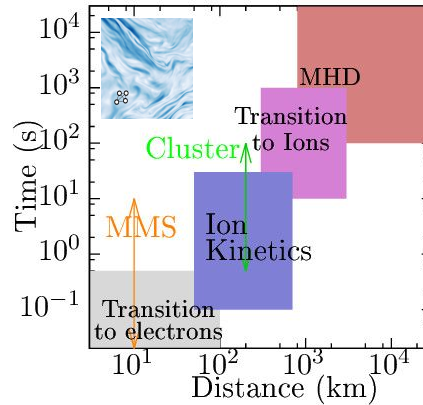
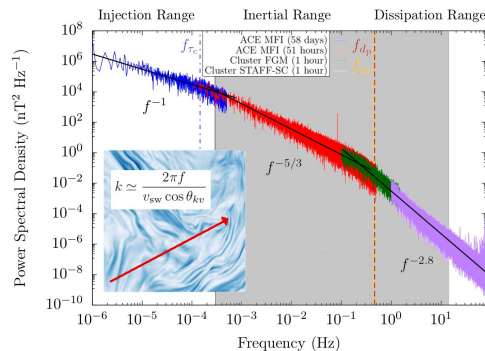


Multi-spacecraft missions (e.g. Cluster and MMS) revolutionized our understanding of the spatial structure of some plasma processes but are limited to measuring only a single scale size at a time.





Turbulence is Multiscale; A **Cross-Scale** Observatory is Needed



To transform our understanding of turbulence, we need simultaneous measurements at many points, with separations between the spacecraft spanning MHD and ion scales, and the transition region in between.

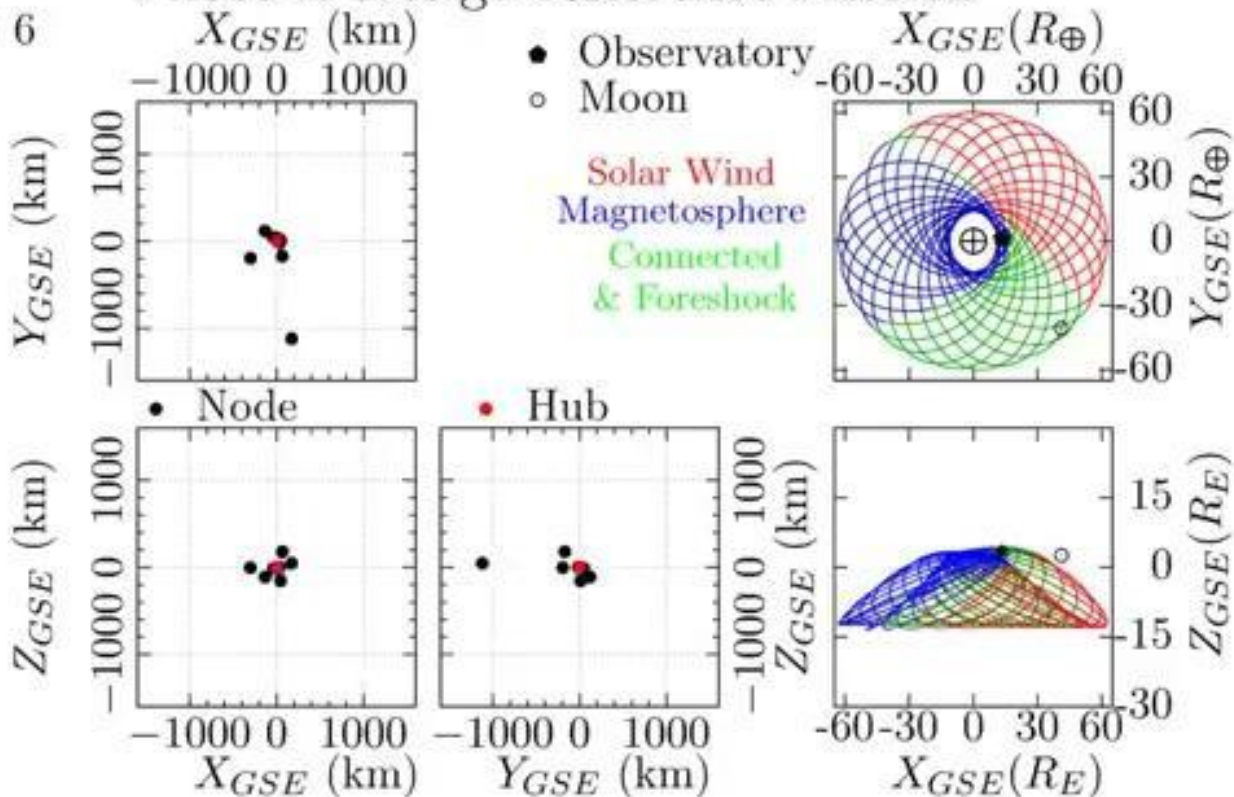


Characterizing Turbulence with a Multipoint Observatory



The HelioSwarm Observatory: Phase-B Design Reference Mission

Hour 6



HelioSwarm is a **nine spacecraft** Heliophysics Medium Class Explorer designed to explore the dynamic 3D mechanisms controlling plasma turbulence.

HelioSwarm is in Phase B, finalizing requirements definitions, with mission PDR in August 2026, with Launch Readiness in 2030.

Trajectory Videos are available here:





Measuring Plasma Turbulence



At fluid scales, the plasma is controlled by the **Momentum Eqn:**

$$\frac{\partial \mathbf{U}}{\partial t} = -\mathbf{U} \cdot \nabla \mathbf{U} - \frac{1}{\rho_0} \nabla \left(p + \frac{B^2}{2\mu_0} \right) + \frac{\mathbf{B} \cdot \nabla \mathbf{B}}{\mu_0 \rho_0}$$

and the **Induction Eqn:**

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{U} \times \mathbf{B})$$

Defined the Elsasser variable, $\mathbf{z}^\pm = \mathbf{U}_\perp \pm \frac{\mathbf{B}_\perp}{\sqrt{\mu_0 \rho_0}}$ we can write a simplified evolution equation:

$$\frac{\partial \mathbf{z}^\pm}{\partial t} \mp (\mathbf{v}_A \cdot \nabla) \mathbf{z}^\pm + (\mathbf{z}^\mp \cdot \nabla) \mathbf{z}^\pm = -\nabla p / \rho_0$$

describing the **propagation of Alfvén waves** and the **nonlinear shearing** that enables the turbulent cascade.

To test theories of turbulence, we need to see how $\mathbf{z}$ changes as a function of scale:

$$\delta \mathbf{z}^\pm(\mathbf{x}, \mathbf{l}) = \mathbf{z}^\pm(\mathbf{x} + \mathbf{l}) - \mathbf{z}^\pm(\mathbf{x})$$

We need to measure density, velocity, pressure, and the magnetic field, and many points simultaneously

HelioSwarm Measures Plasma Density, Velocity, Temperature, and Magnetic Fields



Science Instruments

High-TRL, high-heritage instrument suite optimized for solar wind turbulence measurements.

HUB & NODE

Fluxgate Magnetometer (MAG)

- Vector DC magnetic fields
- Solar Orbiter post-environmental heritage and JUICE design heritage



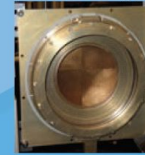
Search Coil Magnetometer (SCM)

- Vector AC magnetic fields
- JUICE design heritage



Faraday Cup (FC)

- Solar wind plasma density and velocity
- Parker Solar Probe, WIND, DSCOVR flight heritage



HUB ONLY

Ion Electrostatic Analyzer (iESA)

- Ion velocity distributions
- Solar Orbiter post-environmental heritage and MAVEN flight heritage



NORTHROP GRUMMAN



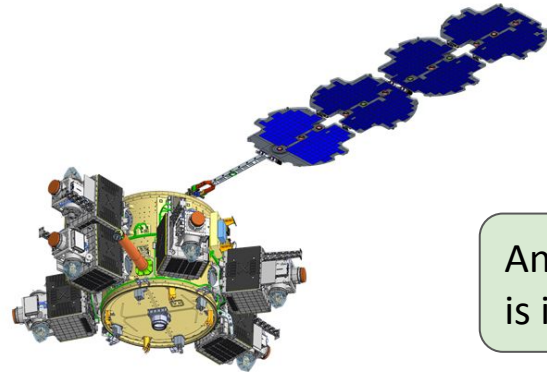
Tim Horbury

Imperial College London/
UKSA

Olivier Le Contel
LPP/CNES

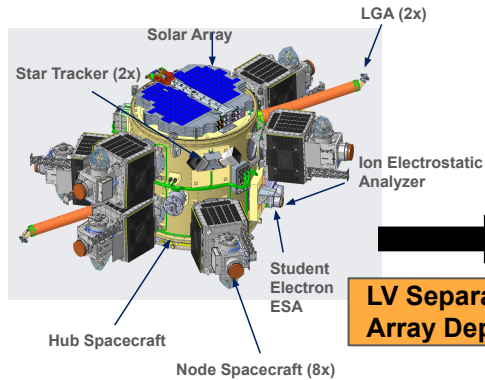
Mike Stevens
Smithsonian
Astrophysical
Observatory

Benoit Lavraud
IRAP/LAB/CNES



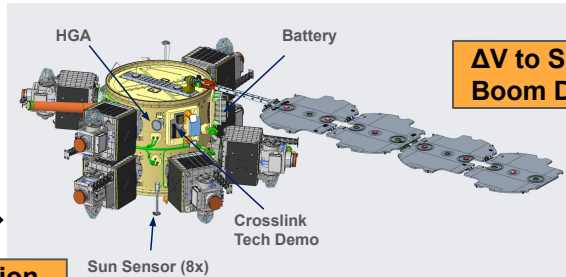
An Student Electron Instrument, lead by **Phyllis Whittlesey** (UC Berkeley), is included as a Student Collaboration Option for installation on the Hub.

Launch Configuration



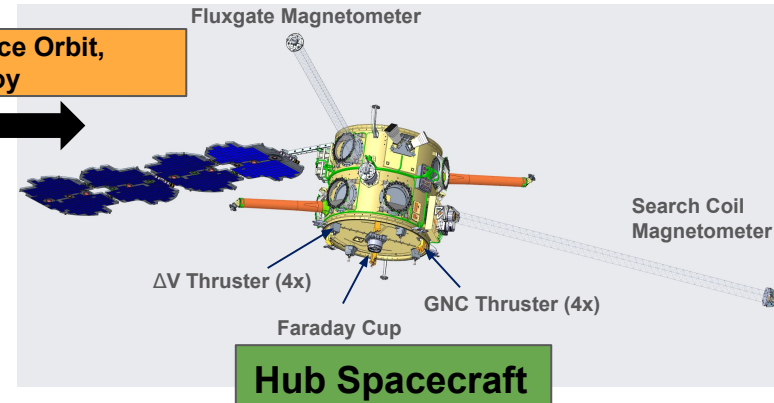
LV Separation, Array Deploy

Cruise & Commissioning Configuration

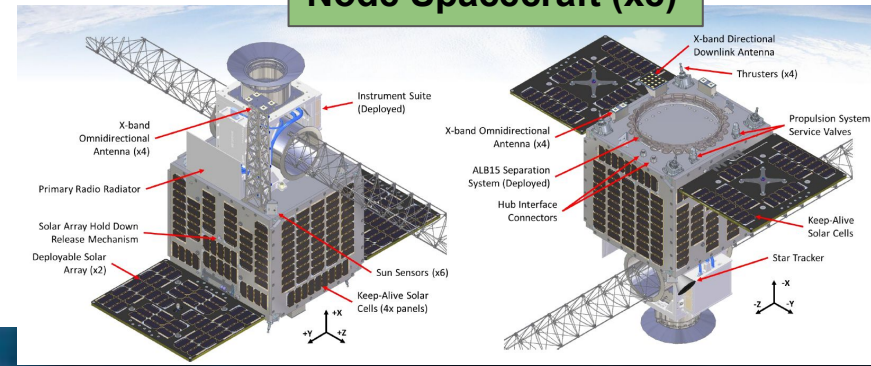


Node Separations

Science Configuration



Node Spacecraft (x8)



HS launches as a combined system. The Hub transports the eight Nodes to the Science Orbit, where they are deployed into designated loops to create the required geometries.



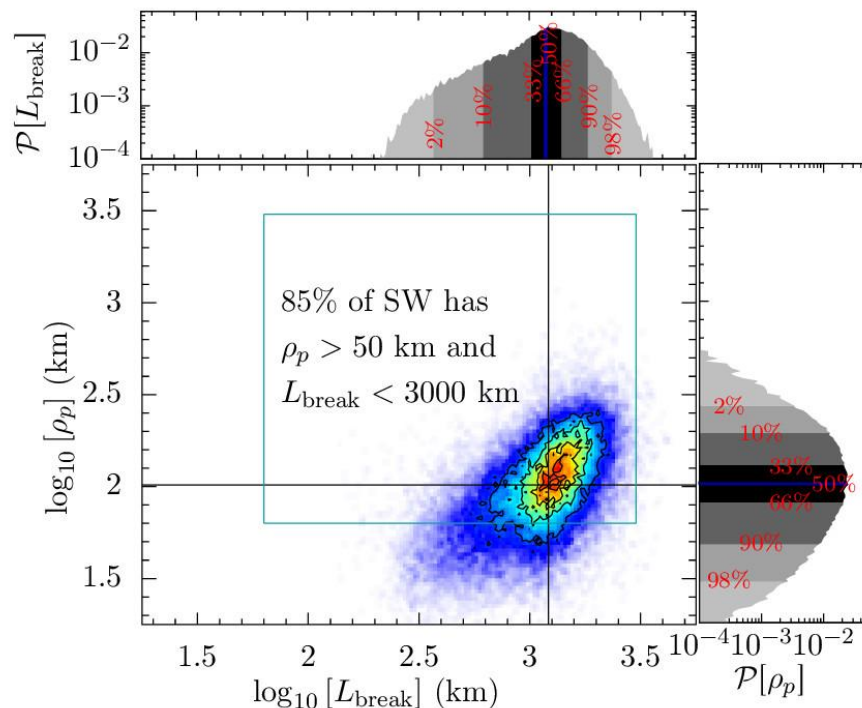
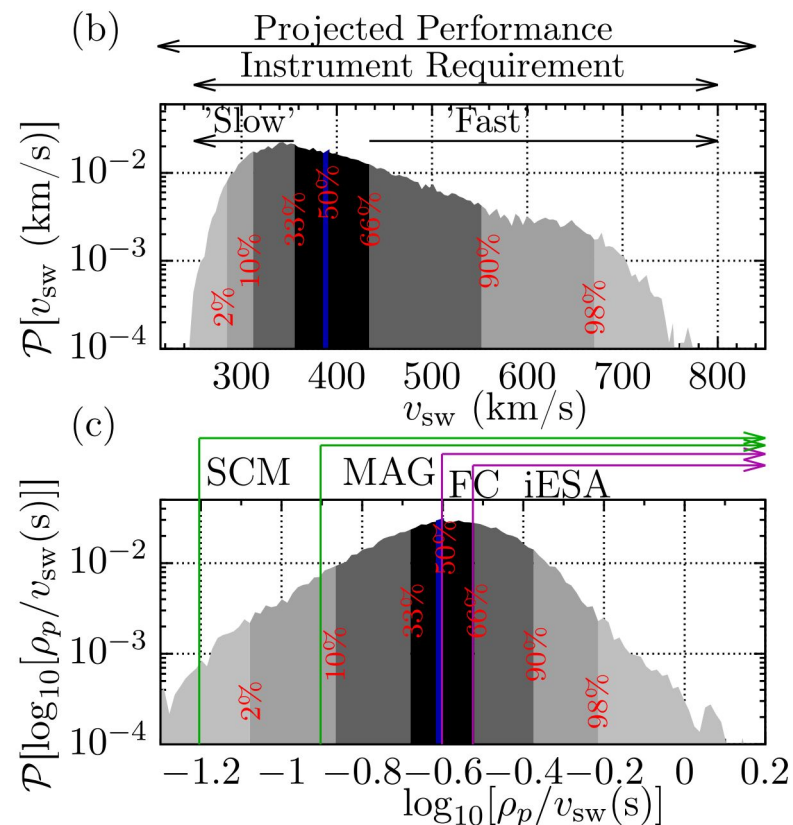
HelioSwarm's Hub and eight nodes measure MHD and ion-scale turbulence in near-Earth space plasma

The nodes are **not** in orbit around the hub, but all spacecraft orbit around the Earth.

The trajectories are designed so the inter-spacecraft separations satisfy the required geometric configurations during each two week cislunar orbit.



HS Spans MHD and Ion-Scale Temporal and Spatial Structures

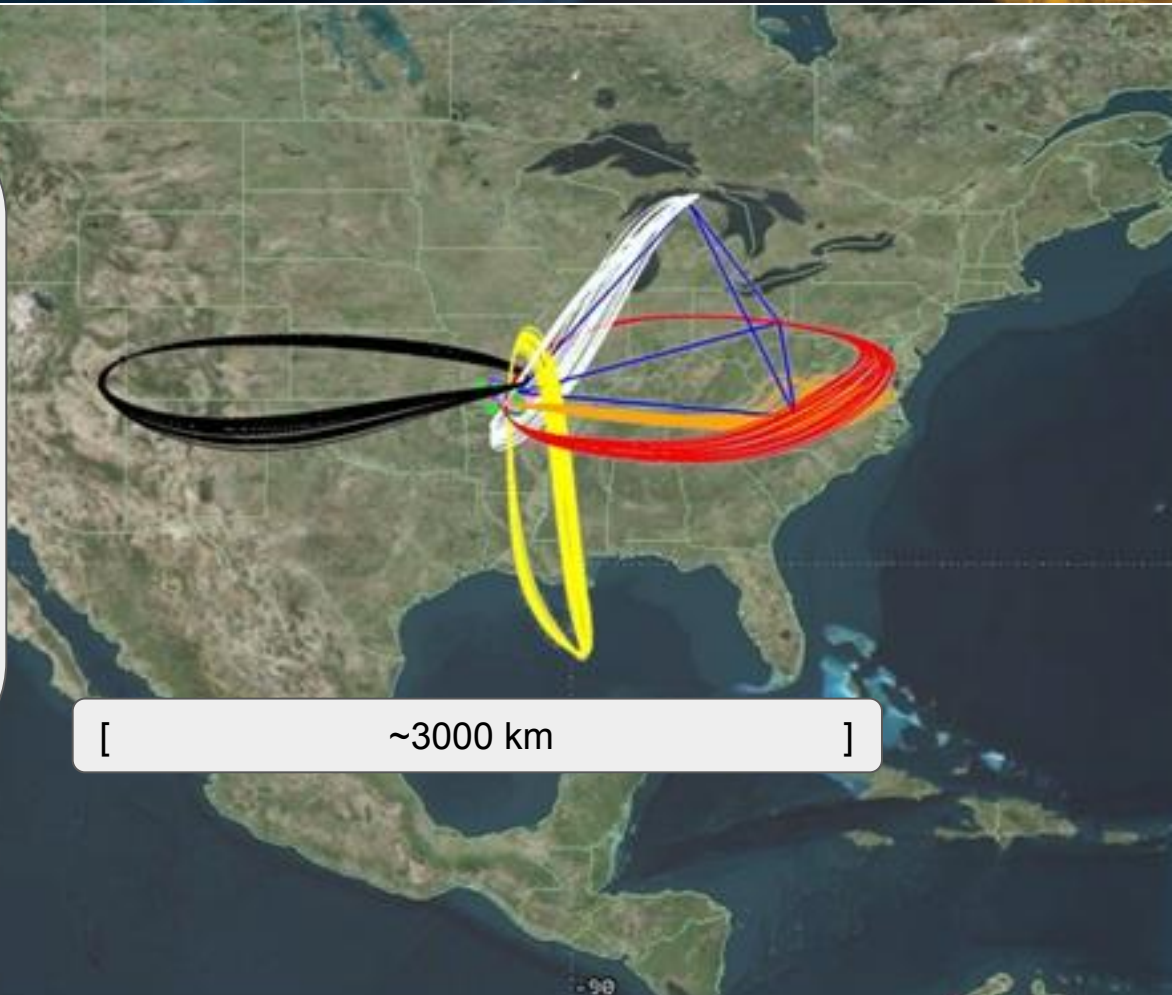


Decades of Solar Wind Observations dictate necessary cadences and spatial separations.

A Comparative Sense of Scale

The smallest separations are on the scale of 10 km.

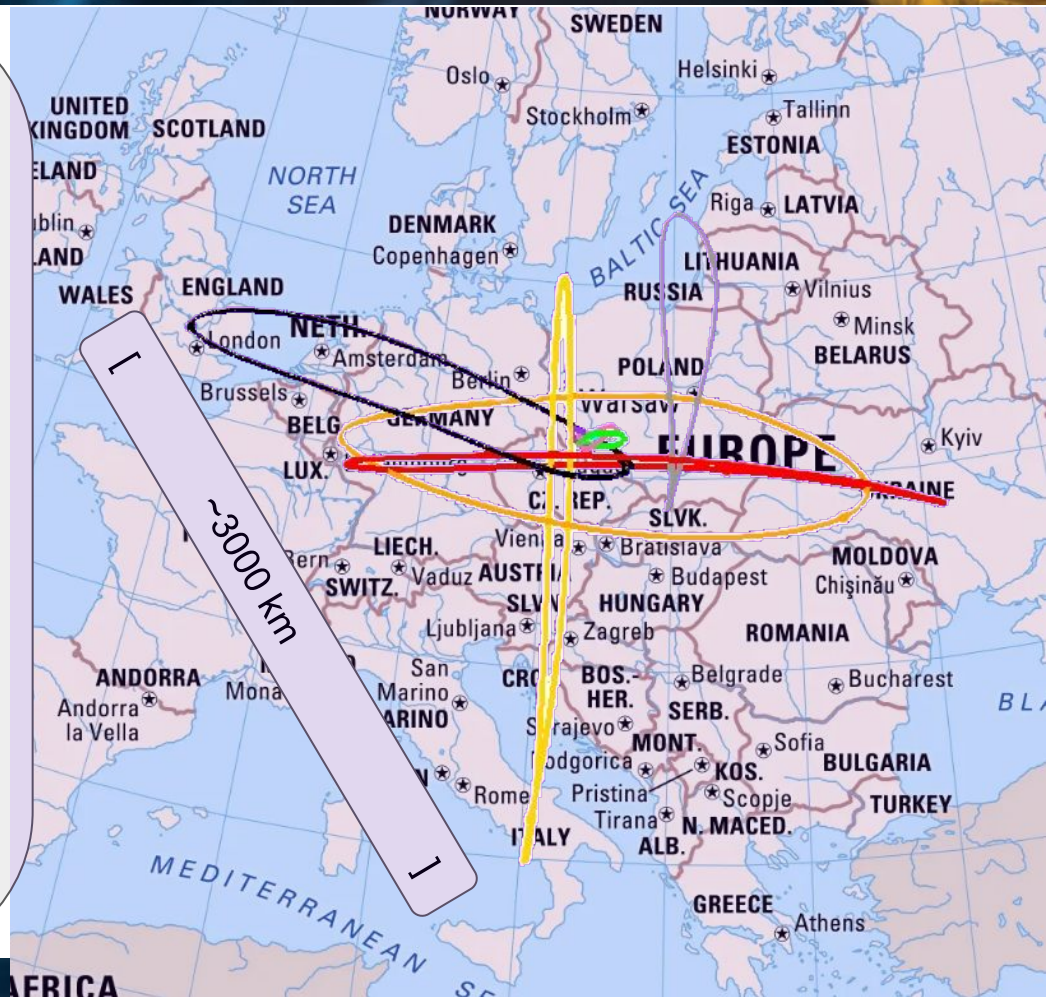
The largest dimensions of the swarm are 1000's of km, approximately equivalent to the distances between the Rocky Mountains and the Atlantic Coast, and the Canada's Southern border to the launch site.



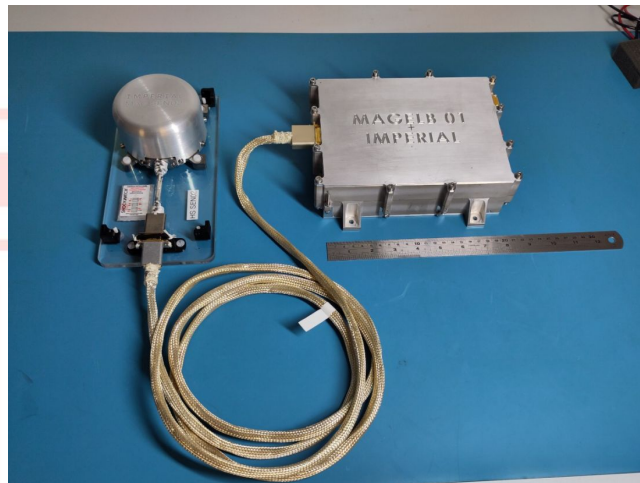
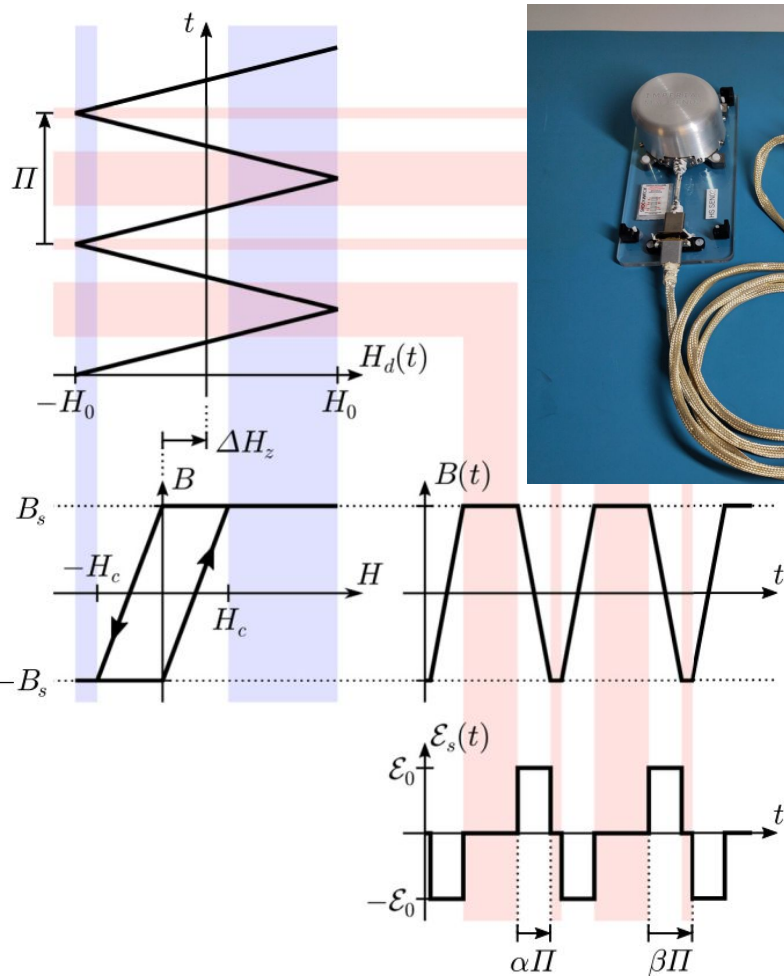
A Comparative Sense of Scale

The smallest separations are on the scale of 10 km.

The largest dimensions of the swarm are 1000's of km, approximately equivalent to the distances between Italy and the UK.



Fluxgate Magnetometer (MAG; T. Horbury)



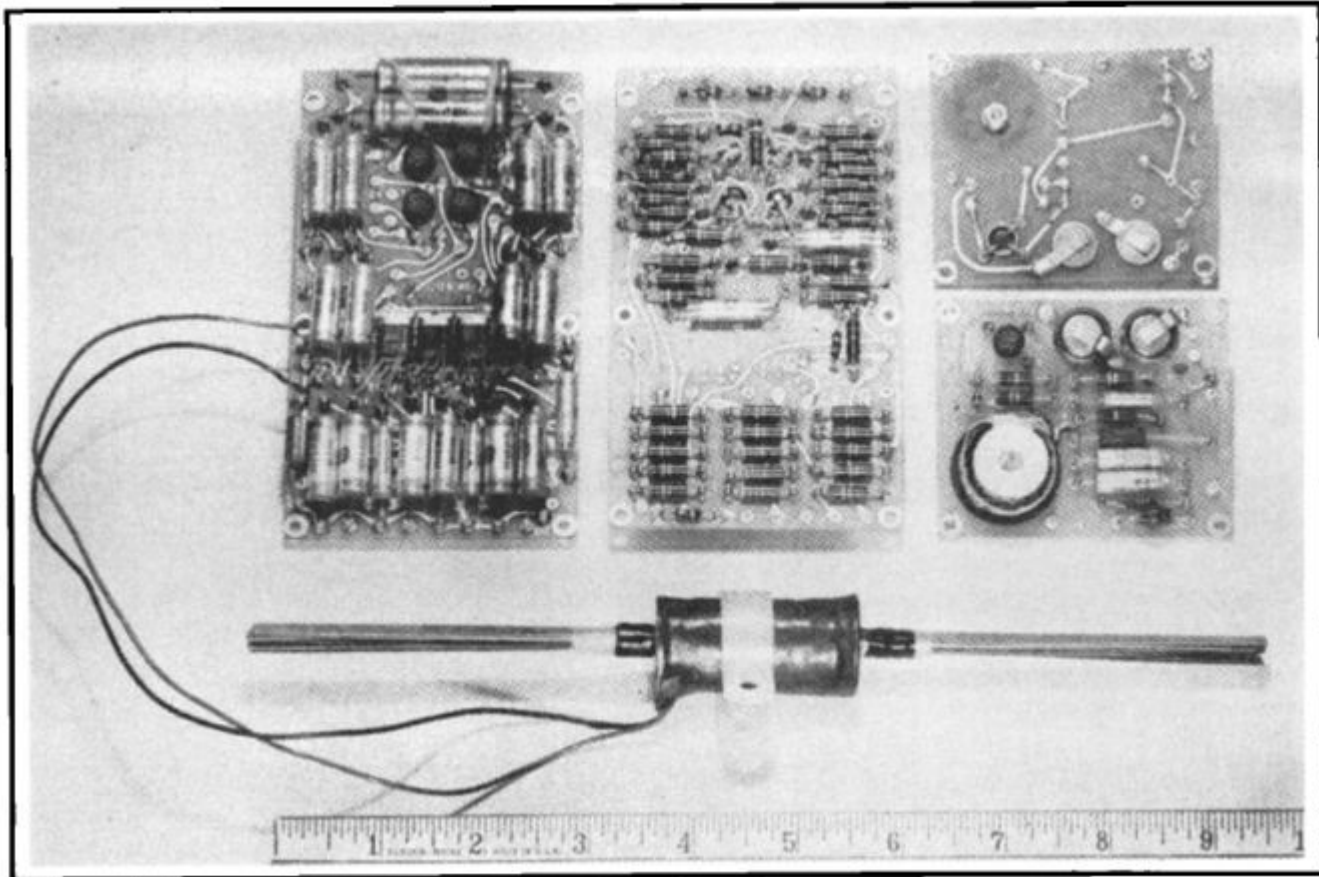
A fluxgate magnetometer relies on the hysteresis of ferromagnetic materials (left).

The magnetic field B inside the material depends on the auxiliary field H applied and the history of the core's magnetization.

This field inside the material can be converted into a voltage, which arises as an irregular square wave, which can be converted with knowledge of the core and windings into a measured field.



Search Coil Magnetometer (SCM; O. LeContel)

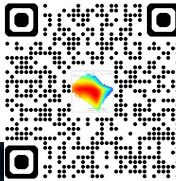


A SCM is a coil of wire that wraps around a portion of a core made from a high-permeability material, which serves to amplify the magnetic field.

The measured voltage will depend on the geometry of the core and number of turns in the coil.

$$\mathcal{E}(t) = \frac{2\pi v \mathcal{N} A \mu_c B_{0,z}}{c} \sin(2\pi vt + \phi).$$

As a single coil can only detect fluctuations in the component parallel to the coil's axis, SCMs include three orthogonal coils to enable measurements of the vector magnetic field.



Why do we need two magnetometers?

A SCM's sensitivity scales linearly with frequency, are thus mostly used in the frequency range from a few Hz to several kHz. Although the MAG is more complex than the search-coil magnetometer, it is much better suited to measuring the background magnetic field and low-frequency (10 Hz) magnetic fluctuations, and has poor sensitivity to fluctuations around or above the frequency of its drive coil.

HS carries both an SCM and MAG in order measure from the DC field to ion-scale frequencies characteristic of the onset of turbulent dissipation. These signals can be carefully merged to create a single magnetic field data product.

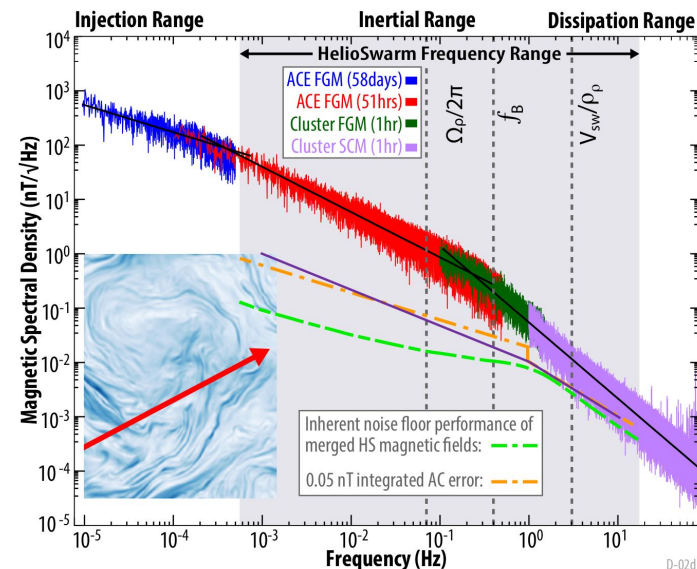
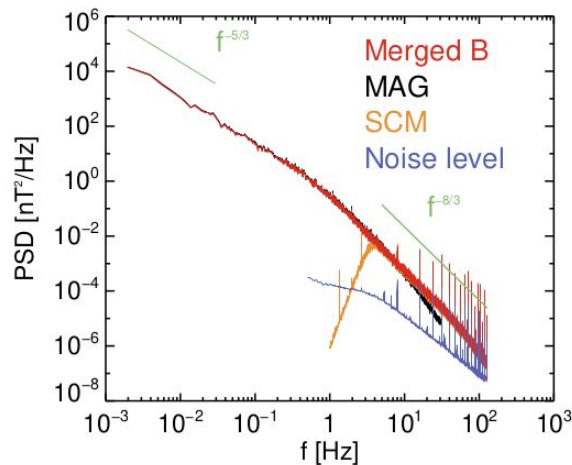
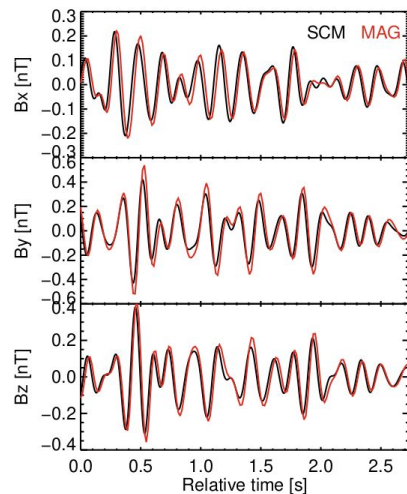
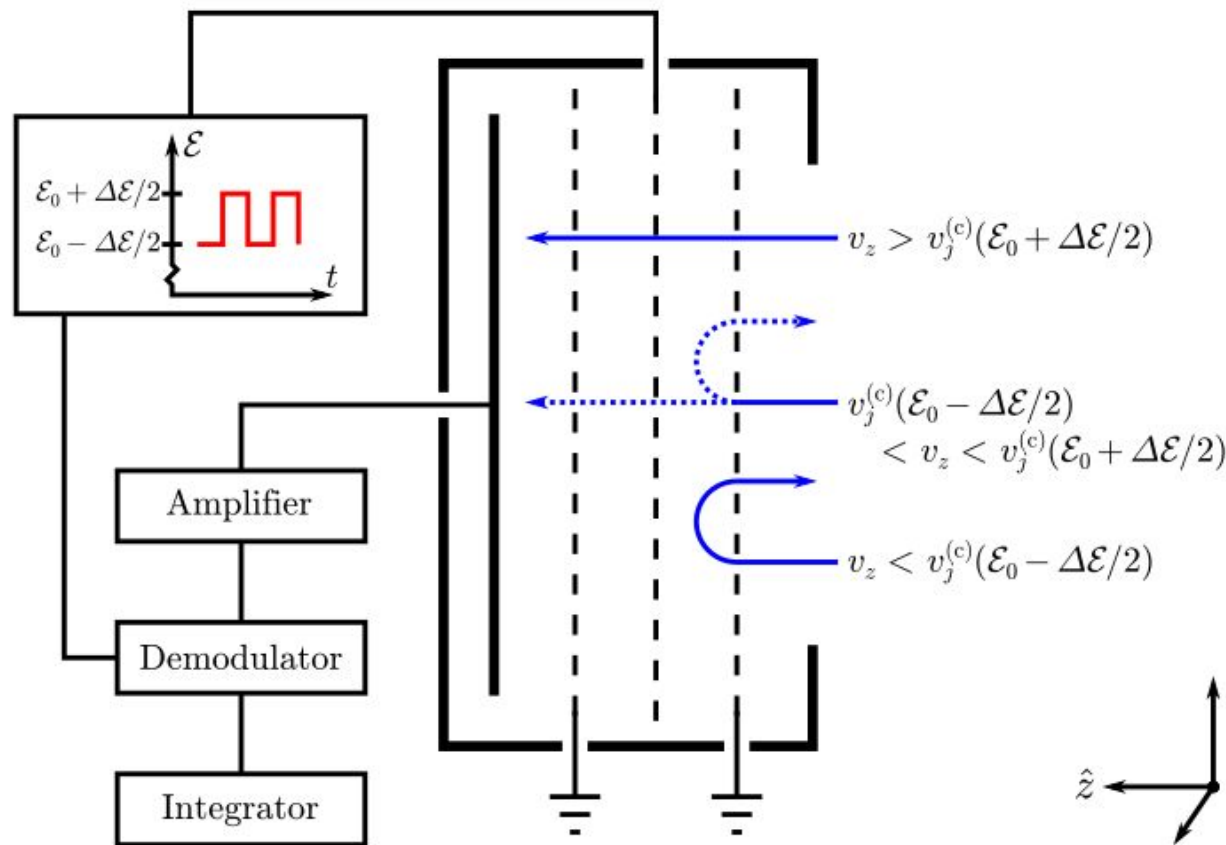


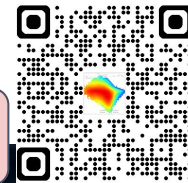
Fig. 1. MAG (red) and SCM (black) waveforms in SRF frame, filtered between 3.5 Hz and 8 Hz on September 6 2022 around 15:13:28.

Faraday Cup (FC, M. Stevens)



A FC consists of a grounded metal structure with an aperture, mesh grids, and collector. Applying a voltage is to the middle grid, only charged particles with narrow range of energy are allowed to pass to the collector. By varying the offset voltage, a reduced velocity distribution is measured, enabling the rapid determination of **density** and **vector velocity**.

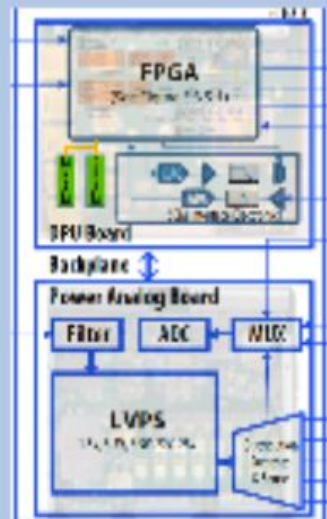
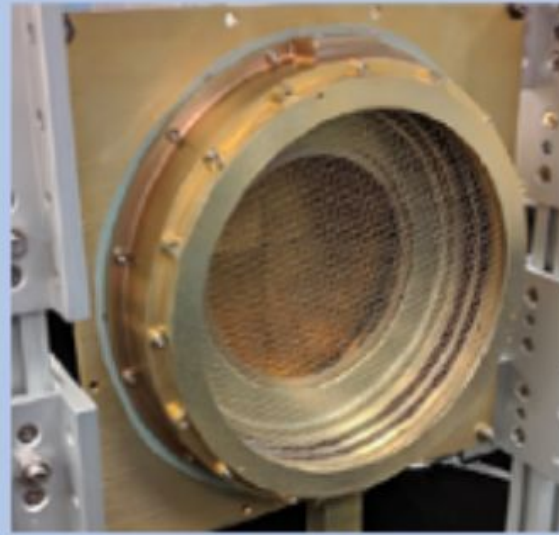
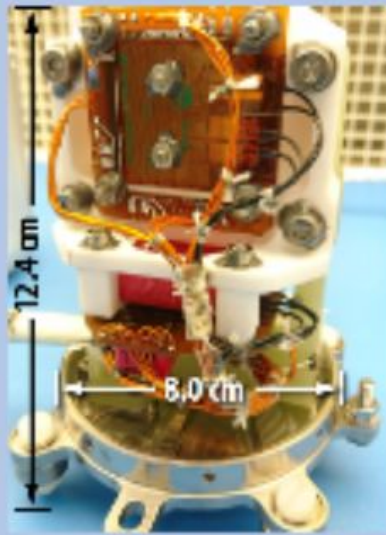
A detailed overview of space plasma instrumentation can be found in [Verscharen, Klein, Maruca 2019](#), Section 2



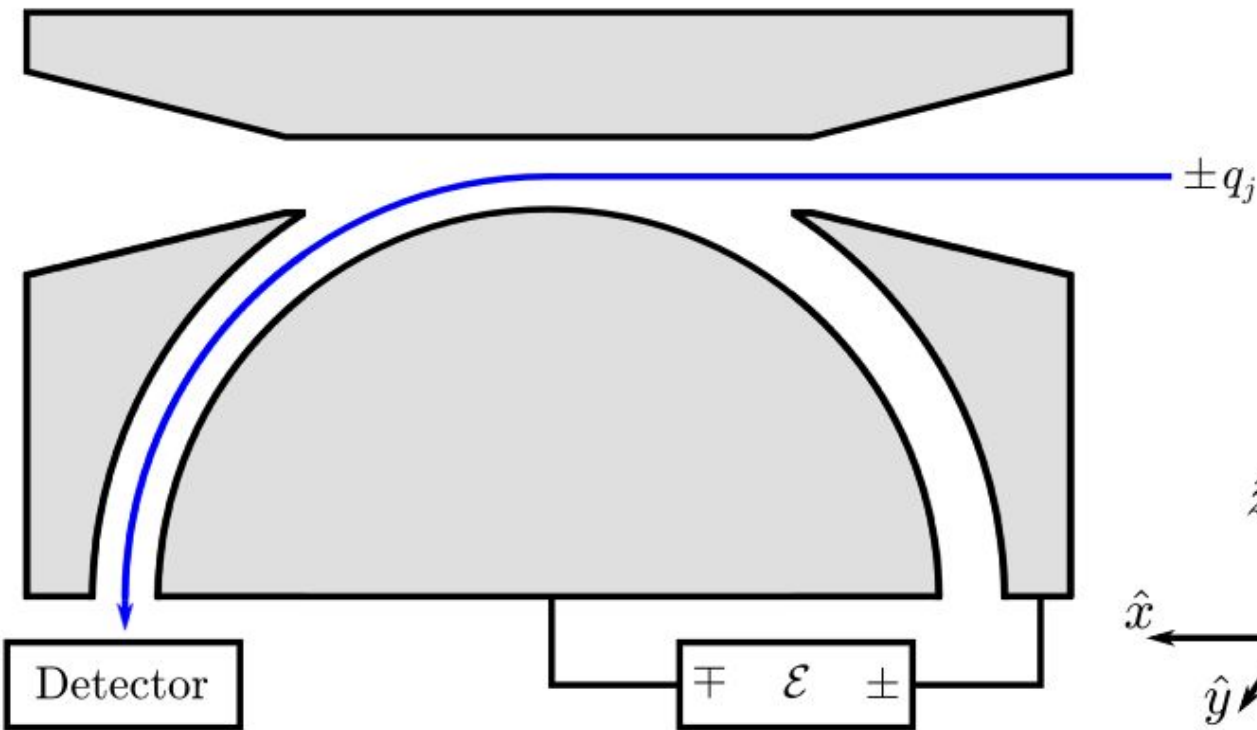
The SCM, MAG, and FC are present on all spacecraft. Combined they enable the calculation of the Elsasser variable

$$\mathbf{z}^{\pm} = \mathbf{U}_{\perp} \pm \frac{\mathbf{B}_{\perp}}{\sqrt{\mu_0 \rho_0}}$$

The three instruments are controlled by an IDPU, through which all of their measurements are digitized, acting as a single interface for the instrument suite.



Ion Electro Static Analyzer (iESA; B. Lavraud: Hub Only)

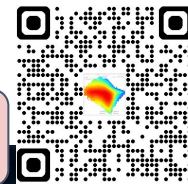


Particles enter the top of the larger hemisphere and are subjected to the electric field.

The field strength and instrument curvature define an energy-per-charge range for particles to reach the detectors at the base of the hemispheres.

Through careful positioning of the anodes, the iESA builds a full distribution as a function of energy, elevation, and azimuth, providing a more detailed VDF than the FC.

A detailed overview of space plasma instrumentation can be found in [Verscharen, Klein, Maruca 2019](#), Section 2



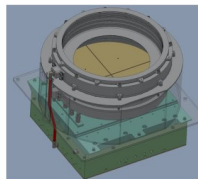
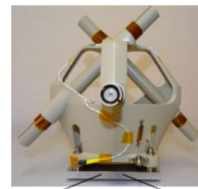
Observable Quantities and Cadences

Zhao et al 2021 (SoLo)

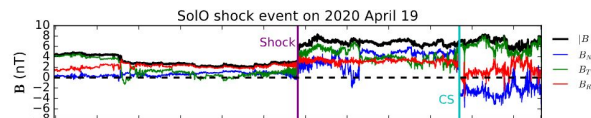
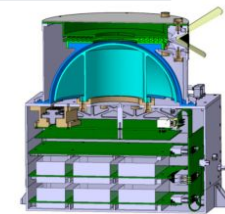


Observable	Requirement	Projected Performance	Instrument
Multi-point vector DC IMF $\mathbf{B}$	DC to 2-Sps ± 100 nT 0.15 nT per axis	DC to 16-Sps ± 128 nT 0.1 nT per axis	MAG (all SC)
Multi-point vector AC IMF $\mathbf{B}$	0.1 to 32-Sps 15/1.5 pT/ $\sqrt{\text{Hz}}$ at 1/10 Hz	up to 32-Sps 6/0.6 pT/ $\sqrt{\text{Hz}}$ at 1/10 Hz	SCM (all SC)
Multi-point proton density n_p	0.15 s 0.2-20 cm^{-3} $\pm 6\%$	0.125 s 0.1-50 cm^{-3} $\pm 5\%$	FC (all SC)
Multi-point proton velocity $\mathbf{v}_p$	0.15 s 250-800 km/s $\pm 3\%$	0.125 s 212-840 km/s Accuracy $\pm 1\%$	FC (all SC)
Single-point proton temperature T_p	0.3 s 10^4 - 5×10^5 K $\pm 5\%$	0.15 s 10^4 - 10^6 K $\pm 1.8\%$	iESA (Hub)
Single-point temperature anisotropy $\frac{T_{\perp}}{T_{\parallel}}$	0.3 s 0.2-5 $\pm 6\%$	0.15 s 0.1-10 $\pm 3.4\%$	iESA (Hub)
Single-point α -proton density ratio $\frac{n_{\alpha}}{n_p}$	Hourly Averages 0-40% $\pm 10\%$	10 s 0-100% $\pm 3.4\%$	iESA (Hub)

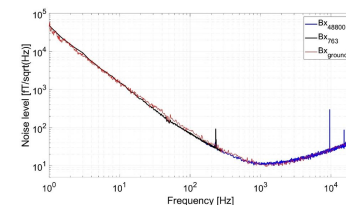
All S/C



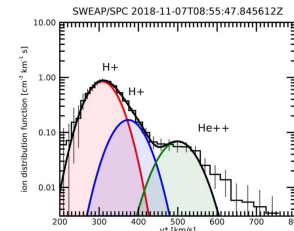
Hub Only



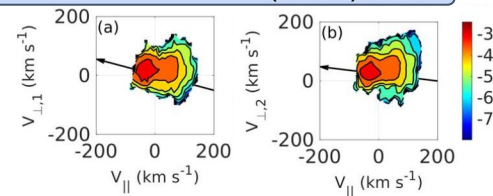
Wahlund et al 2024 (JUICE)



Case et al 2020 (PSP)



Perrone et al 2024 (SoLo)



(G1) Reveal the 3D spatial structure and dynamics of turbulence in a weakly collisional plasma.

G1O1: Reveal how turbulence energy transfers in the typical SW plasma as a function of scale and time.

G1O2: Reveal how the turbulent cascade of energy varies with background parameters in different SW environments.

G1O3: Quantify the transfer of turbulent energy between fields, flows, and proton heat.

G1O4: Identify the thermodynamic impacts of intermittent structures on protons.

(G2) Ascertain the mutual impacts of turbulence, variability, and boundaries near large scale structures.

G2O1: Determine how SW turbulence affects and is affected by large-scale structures such as Coronal Mass Ejections (CMEs) and Corotating Interaction Regions (CIRs).

G2O2: Determine how driven turbulence differs from that in undisturbed SW.



(G1) How is Turbulence Structured? How does it Evolve?

G1O1: What is the (anisotropic) cascade rate?

G1O2: How does the cascade rate change with system parameters?

G1O3: How does energy move from flows to hotter plasma?

G1O4: Do current sheets interrupt the cascade? What role does dynamic alignment play?

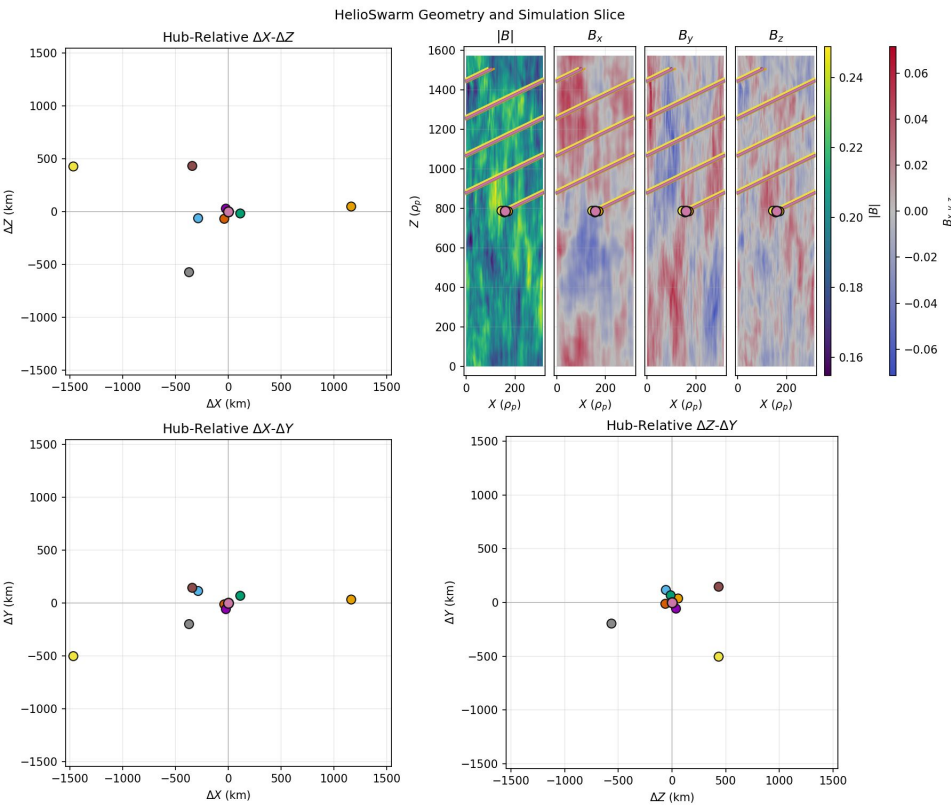
(G2) How do Boundaries and Structures Reshape Turbulence?

G2O1: Do eruptions from the Sun restructure days-old turbulence?

G2O2: How does turbulent structures change as it passes through the magnetosphere?



“N Spacecraft are Better, if the Locations are *Judiciously Selected*”



A key set of HS requirements are on the inter-spacecraft geometries, creating the necessary 3D structures to perform the various analysis methods discussed this week.

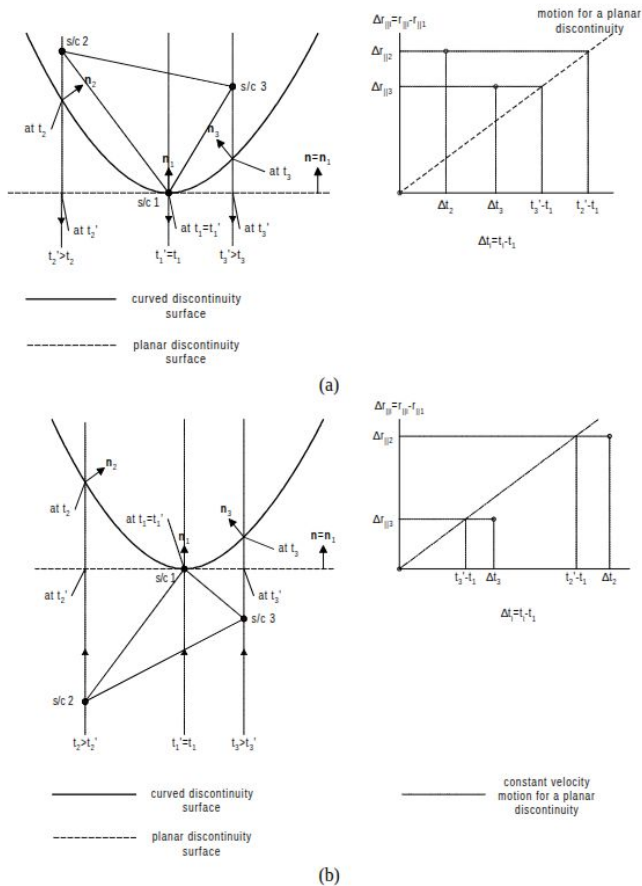
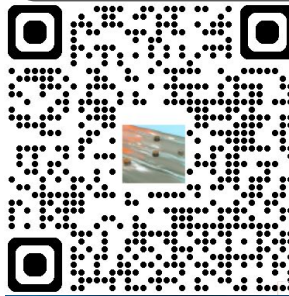


Figure 11.7: The analysis of a non-planar boundary structure.

In preparation for the Cluster launch, an omnibus of analysis methods for constellations of more than one spacecraft was constructed.

In addition to discussions underlying the mathematics of calculating spatial gradients and discontinuity analysis across multiple points, they also offer an excellent overview of time series analysis and resampling of data.

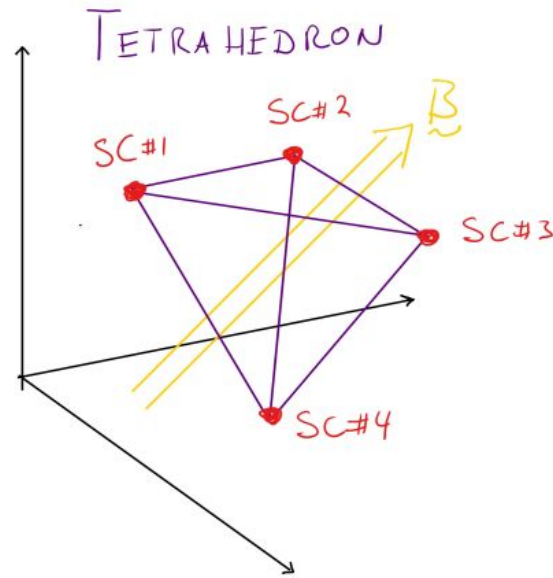
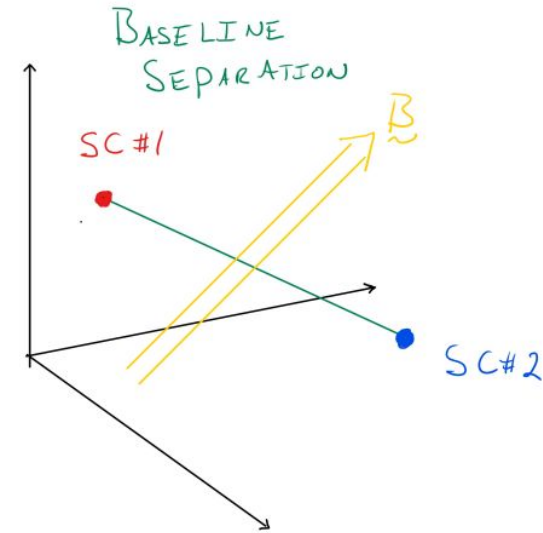
While most of these tools are generalizable to N spacecraft, their main application has been to N=4.



Paschmann & Daly 1998: ISSI SR 001



Observatory Geometric Configurations



N spacecraft yields:
 $\rightarrow N(N-1)/2$ baselines
 $\rightarrow C(N, M) = N! / [M!(N-M)!]$
 polyhedral with M vertices

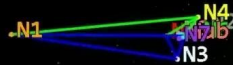
The more baselines and polyhedra the observatory measures, the more scales can be covered, and the more data is produced for statistical analysis methods.

# Spacecraft	4	5	6	7	8	9	10	11
baselines	6	10	15	21	28	36	45	55
tetrahedra	1	5	15	35	70	126	210	330
total polyhedra	1	6	22	64	163	382	848	1816



The Phase B representative trajectories on an hourly cadence are publicly available at
<https://zenodo.org/records/13887062>

Spatial Gradients Require Regular Polyhedra



Chief GSE Axes

Quantifying changes in spatial structure requires measurements of the plasma and magnetic field made at at least four space (i.e. the vertices of **polyhedra**). The canonical example is the **curlometer** method, which enables the measurement of the current from the curl of the magnetic field.

These can be coupled with measurements of the temperature at a single point to quantify the plasma heating.

Wave telescope: Multipoint method for determining the wavevectors of plasma waves and their associated 3D power distributions.

Pressure-strain interaction (Pi-D): Measurement of the dilation, $-(\mathbf{P} \cdot \nabla) \cdot \mathbf{v}$, describing the local conversion between flow and thermal energy.

Curlometer & Gradient Methods: Multipoint methods for constructing current density/intermittent structures from spatially distributed measurements.



Constructing Regular Polyhedra



The observatory geometry can be described by eigenvectors (a,b,c) of the volumetric tensor

$$R = N^{-1} \sum (r_{\alpha} - r_0)(r_{\beta} - r_0)^T$$

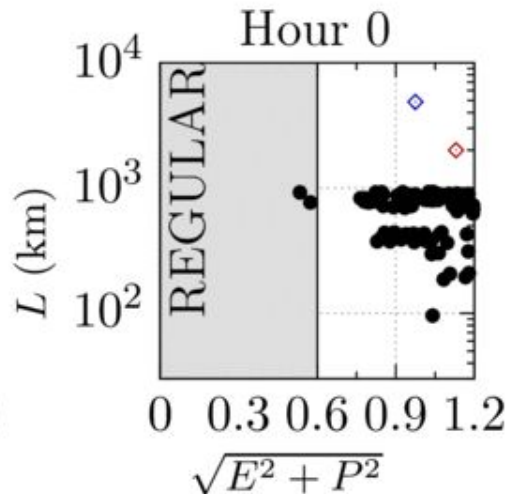
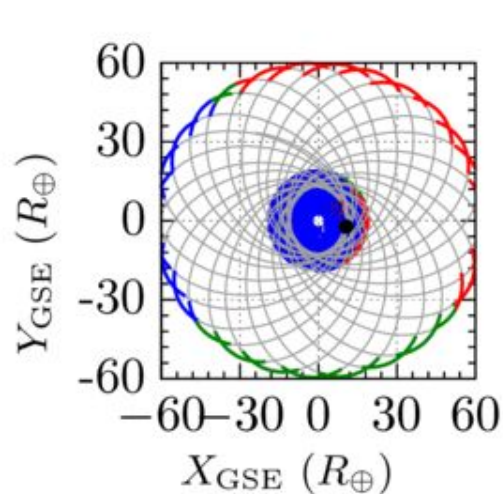
which can be interpreted geometrically as a characteristic **size** $L=2a$ and a **planarity** and **elongation**

$$P = 1 - b/a \text{ and } E = 1 - c/b$$

Smaller values of E & P enable accurate determinations of the structure near scale size L.

The HelioSwarm trajectories are designed so that the regular tetrahedra are of different sizes, so that we can accurately measure the plasma structure at small and large scales simultaneously.

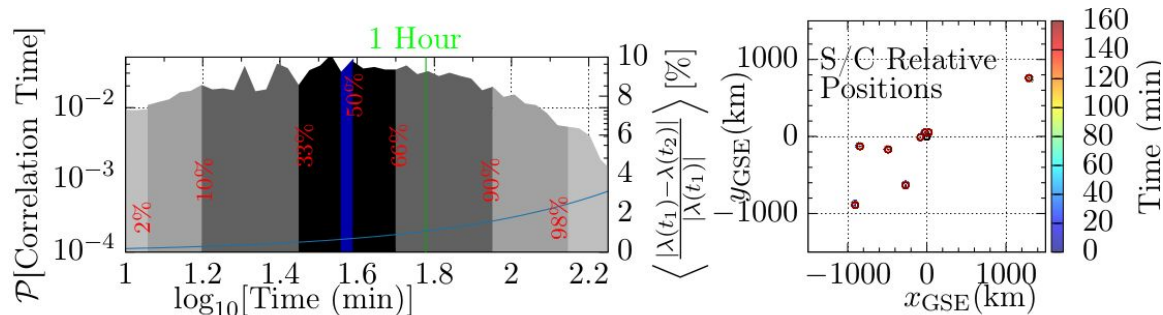
Advantages of a Cislunar Orbit



- HelioSwarm: SRD8A 0x1bc
- ◊ Cluster II: 01/2005-06/2006
- ◊ Cluster II: 01/2002-06/2003

Solar Wind
Magnetosphere
Foreshock

The larger apogee produces more stable good geometric configurations compared to smaller orbits, enabling statistical studies over longer periods of time.



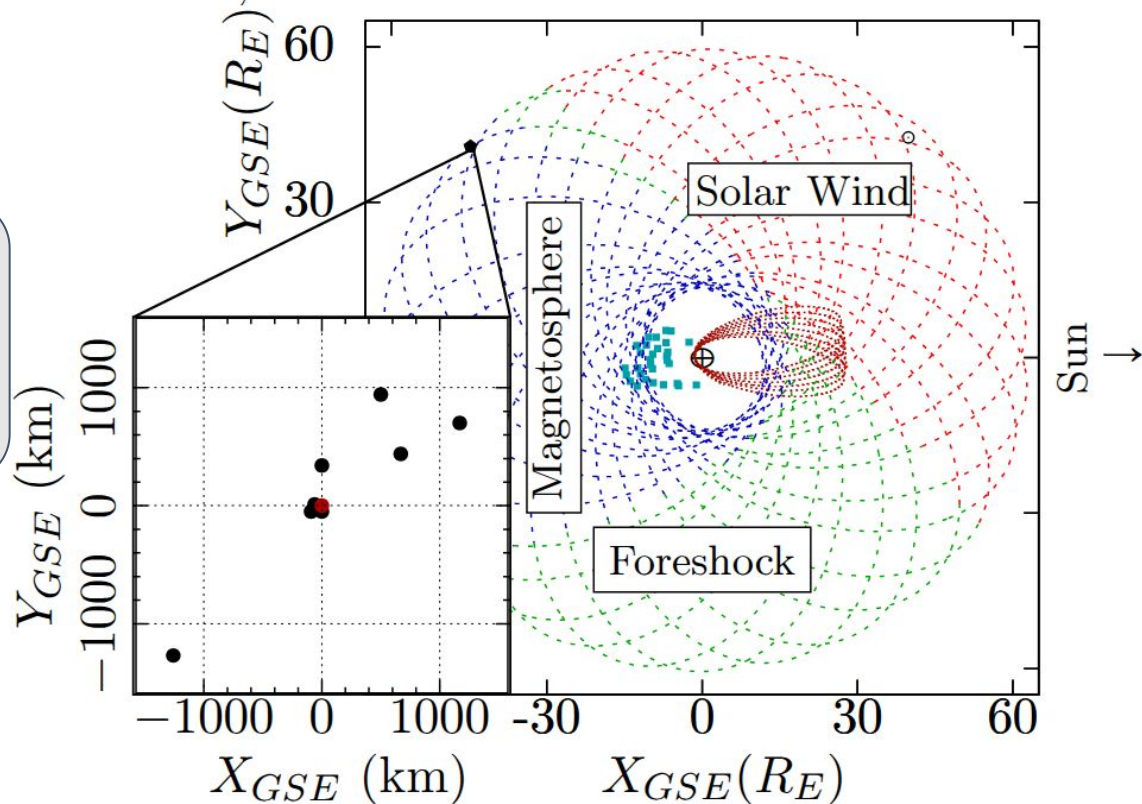


Advantages of a Cislunar Orbit



- Moon
- MagCon (Kepko+2023)
- MMS; Feb. 2023
- HS Orbits
- HS (Klein+2023)

HelioSwarm's Science Orbit places it further from the Earth than previous missions such as Cluster, yielding longer residences in the pristine solar wind.



The Wave Telescope

Given a set of N-spatially distributed (Fourier Transformed) measurements of a field

$$\mathbf{A}(\omega) = \begin{pmatrix} \mathbf{A}(\omega, \mathbf{r}_1) \\ \mathbf{A}(\omega, \mathbf{r}_2) \\ \vdots \\ \mathbf{A}(\omega, \mathbf{r}_N) \end{pmatrix}.$$

We can construct a spatial correlation matrix

$$\mathbf{M}(\omega) = \langle \mathbf{A}(\omega) \mathbf{A}^\dagger(\omega) \rangle.$$

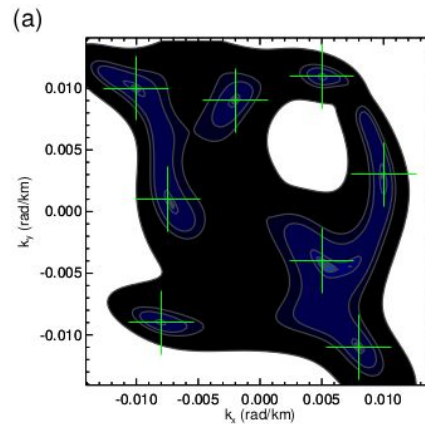
Which combined with the steering vector

$$\mathbf{H}(\mathbf{k}) = \begin{pmatrix} \mathbf{I}_L e^{i\mathbf{k} \cdot \mathbf{r}_1} \\ \mathbf{I}_L e^{i\mathbf{k} \cdot \mathbf{r}_2} \\ \vdots \\ \mathbf{I}_L e^{i\mathbf{k} \cdot \mathbf{r}_N} \end{pmatrix},$$

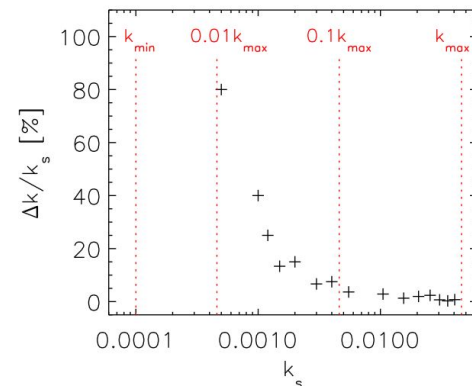
Yields the distribution of power as a function of frequency and wavevector:

$$P(\omega, \mathbf{k}) = \text{Tr} \left\{ \left(\mathbf{H}^\dagger(\mathbf{k}) \mathbf{M}^{-1}(\omega) \mathbf{H}(\mathbf{k}) \right)^{-1} \right\}.$$

With Four Spacecraft, this methods can be applied to accurately reconstruct power associated with several waves simultaneously, but only over a finite range of scales near the characteristic size of the tetrahedra.



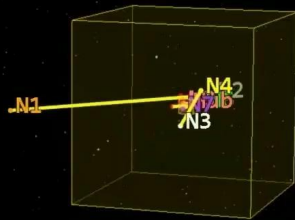
Roberts et al 2014



Sahraoui et al 2010

See Pincon & Lefeuvre 1988, Neubauer & Glassmeier 1990, Tjulin et al 2005, and **tomorrow's lecture...**

Decorrelation Analysis Requires 3D Baselines



Chief GSE Axes

In order to study how the turbulence reshapes itself and how energy is transferred from larger to smaller scales, we need to measure the magnetic field and plasma density and velocity across **baseline separations** that are on fluid scales (>1000's of kms) as well as on scales corresponding to the motions of the protons (<100km) simultaneously.

Cascade Rate: Measure of the transfer of a turbulent fluctuation energy from one spatial scale to another.

2-point correlation: Measure of the temporal and spatial scale over which a spectral element is remade by nonlinear processes.

Structure Functions: Statistics of two-point increments of turbulent fields to reveal scale-dependent, intermittent turbulence.

The HelioSwarm Observatory is designed so that the 36 baseline separations between the 9 spacecraft span the fluid and kinetic scales, as well as the ranges in between.

The trajectories are also designed so that the baselines span a three dimensional space, so we will be able to study differences in the turbulence along and across the direction of the solar wind flow and magnetic field.

# Spacecraft	4	5	6	7	8	9	10	11
baselines	6	10	15	21	28	36	45	55
tetrahedra	1	5	15	35	70	126	210	330
total polyhedra	1	6	22	64	163	382	848	1816



Analysis Approach: Structure Functions

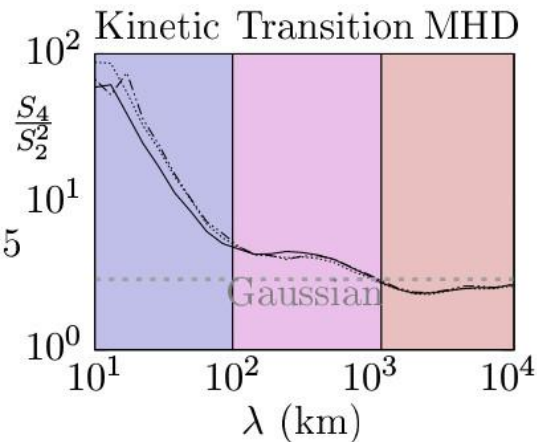
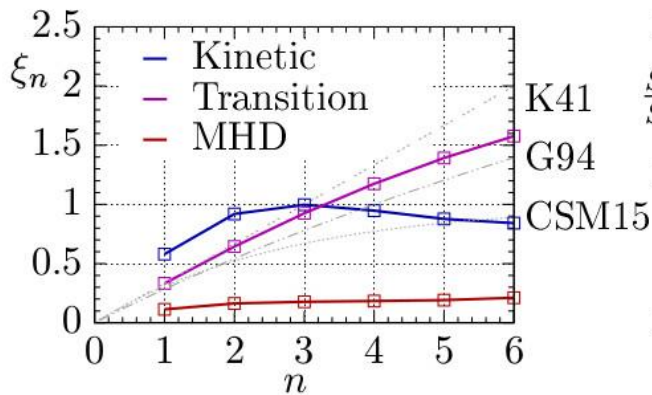
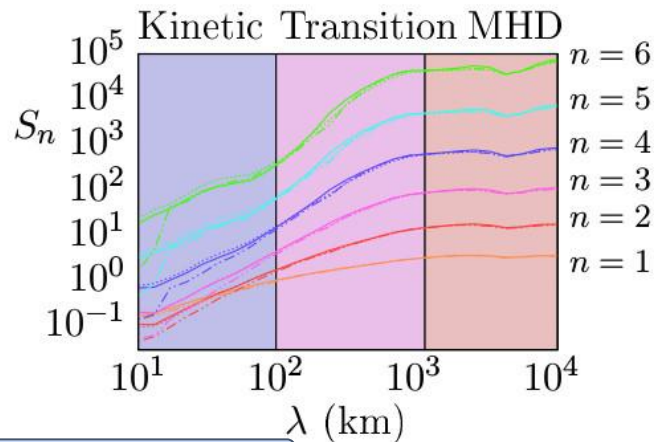


Structure functions characterize the intermittent nature of turbulence by averaging power of two-point increments. With a time series from a single spacecraft, this calculation relies on converting temporal lags into spatial scales:

$$S_n(\lambda = v_{sw}\tau) = \langle [\delta B(t + \tau) - \delta B(t)]^n \rangle$$

With multiple spacecraft, we have orders of magnitude more increments that can be used for this calculation, yielding a more statistically robust coverage of spatial scales with sufficient increment samples to resolve higher orders, and enable comparison to predictions for spectral laws as a function of scale $S_n \propto \lambda^{\xi_n}$

$$S_n(\lambda = \mathbf{x}_i(t + \tau) - \mathbf{x}_j(t)) = \langle \{ \delta B[\mathbf{x}_i(t + \tau), t + \tau] - \delta B[\mathbf{x}_j(t), t] \}^n \rangle$$





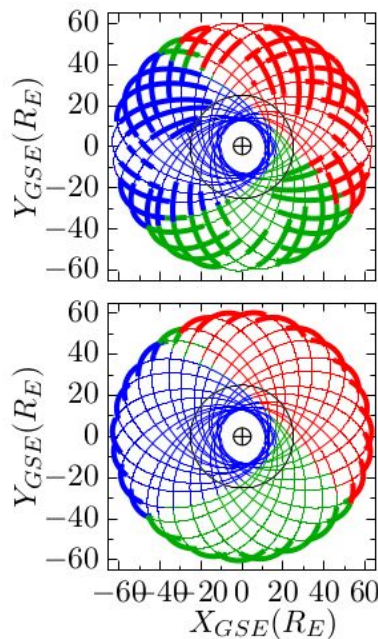
Hours Accumulated and Plasma Environments Measured



HS will accumulate hundreds of hours over a one-year prime science phase in both geometric configurations in the pristine solar wind and magnetospheric regions.

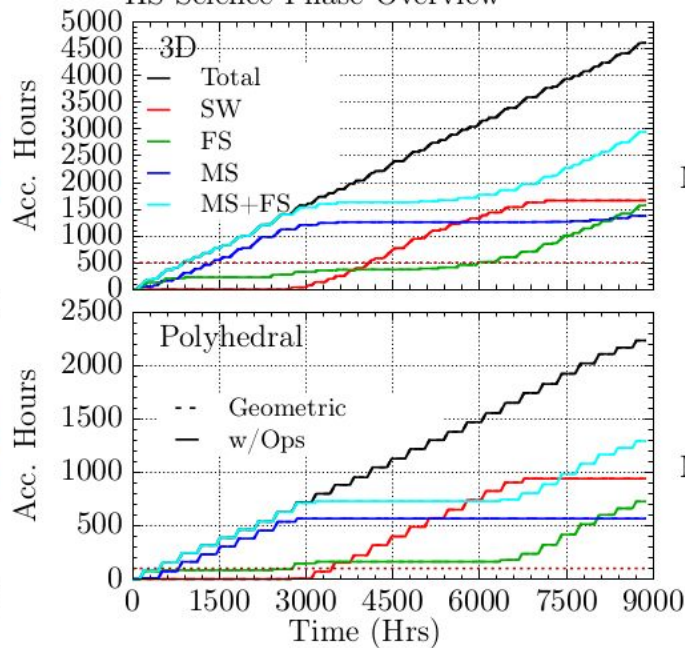
Measurements will be made and downloaded throughout the orbit, regardless of geometric configuration.

HQ-RB-overview



PRERELEASE

HS Science Phase Overview



3D	—	Total
	Ops (Geo.)	Ops (Geo.)
All	- (-)	4618 (4619)
SW	- (-)	1664 (1665)
MS+FS	- (-)	2954 (2954)

Poly.	—	Total
	Ops (Geo.)	Ops (Geo.)
All	- (-)	2233 (2233)
SW	- (-)	940 (940)
MS+FS	- (-)	1293 (1293)



The fluid von Kármán-Howarth expression has been extended to MHD ([Politano & Pouquet 1998](#)) as

$$\frac{\partial}{\partial t} \langle |\delta \mathbf{z}^{\pm}|^2 \rangle + \nabla_l \cdot \langle \delta \mathbf{z}^{\mp} |\delta \mathbf{z}^{\pm}|^2 \rangle - 2\nu \nabla_l^2 \langle |\delta \mathbf{z}^{\pm}|^2 \rangle = -4\epsilon^{\pm}$$

At scales l sufficiently far from injection and dissipation, (c.f. [Adikari et al 2024 PoP](#)) this simplifies to Yaglom's Law:

$$\nabla_\ell \cdot \mathbf{Y}^{\pm} = -4\epsilon^{\pm}, \quad \text{where} \quad \langle \delta \mathbf{z}^{\mp} |\delta \mathbf{z}^{\pm}|^2 \rangle = \mathbf{Y}^{\pm}$$

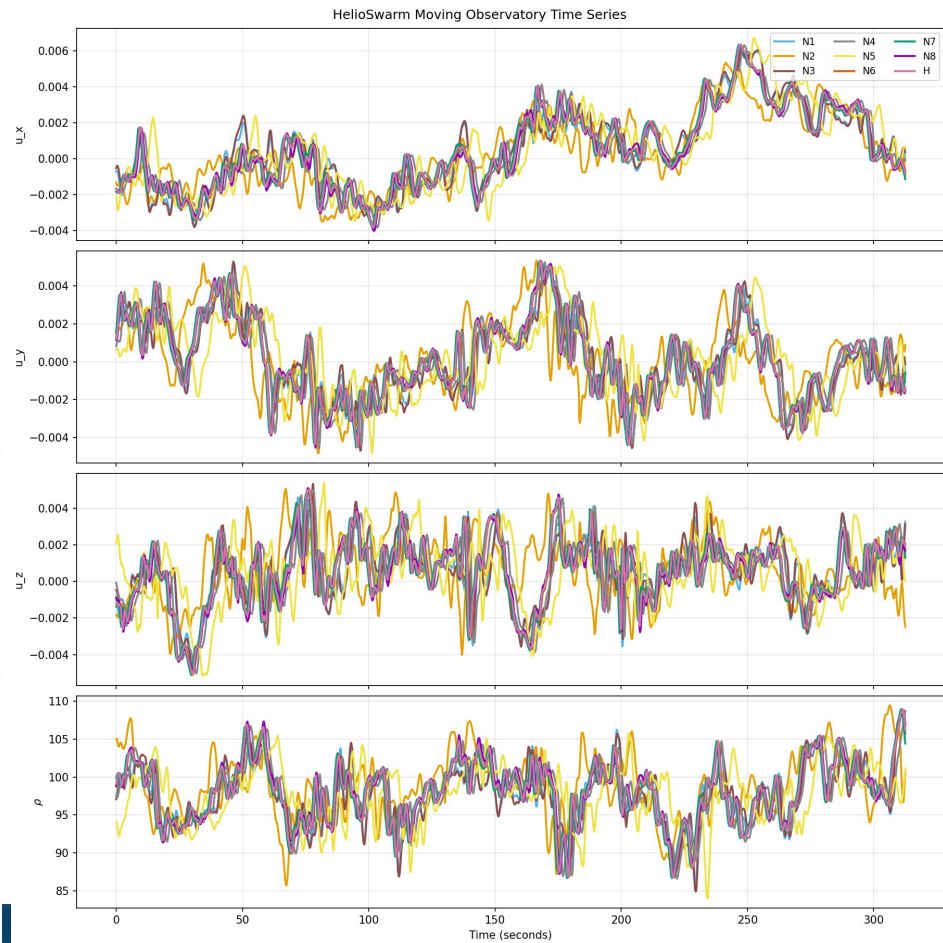
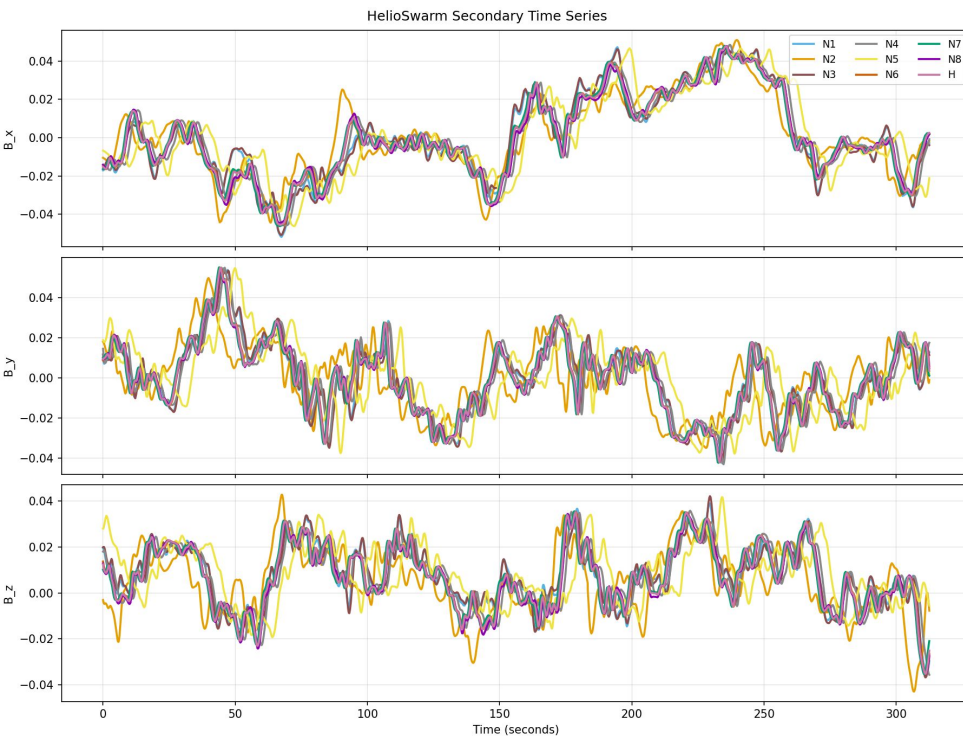
Measurements from all 28 instruments with separations spanning MHD and ion scales combine to enable novel characterizations of multiscale turbulent processes



The Swarm is the Observatory: Plasma Measurements



From an example numerical simulation of turbulence, we sample n , $\mathbf{U}$, and $\mathbf{B}$ along representative HS trajectories.



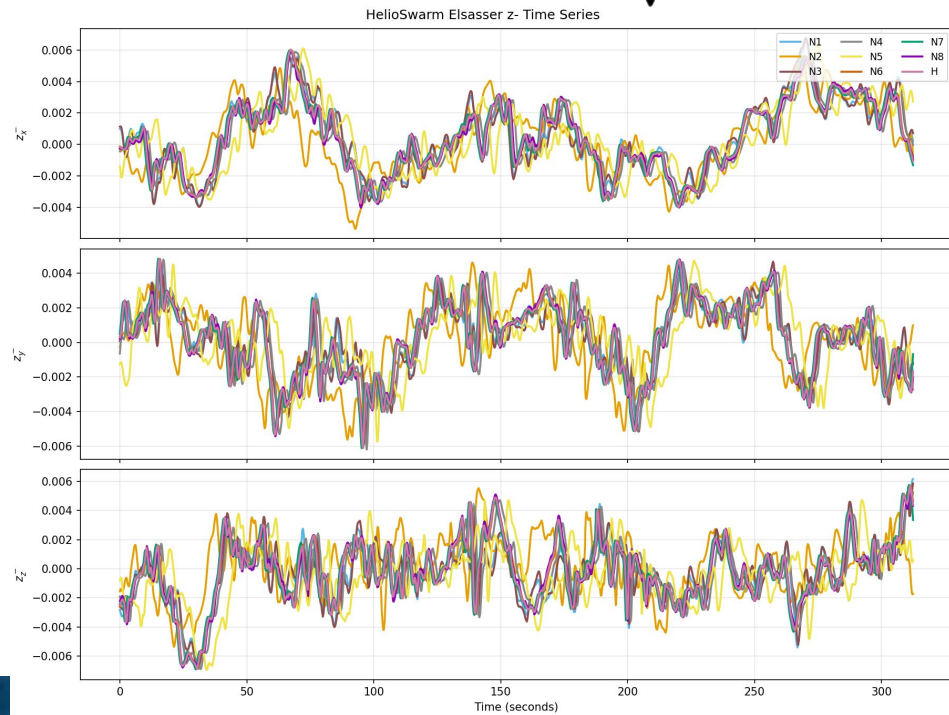


The Swarm is the Observatory: Elsasser Fields



Using the measured $\mathbf{n}$, $\mathbf{U}$, and $\mathbf{B}$, we can construct the Elsasser fields

$$\mathbf{z}^{\pm} = \mathbf{U} \pm \frac{\mathbf{B}}{\sqrt{\mu_0 \rho}}$$





The Swarm is the Observatory: Baseline Separations



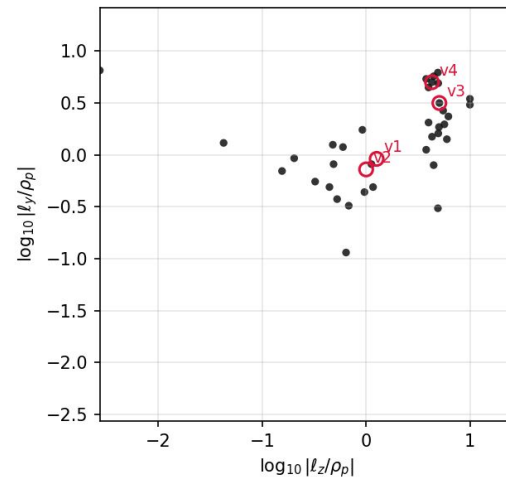
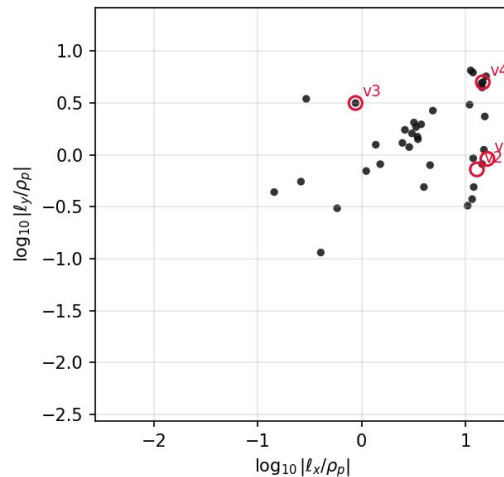
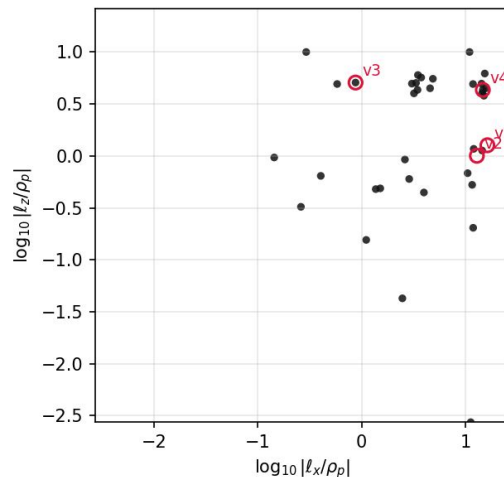
For both structure functions and cascade rates, we need to calculate lagged differences, e.g. :

$$\delta z^{\pm}(x, \ell) = z^{\pm}(x + \ell) - z^{\pm}(x)$$

Given our 9 spacecraft, we will have $9 \times 8 / 2 = 36$ baselines for a given moment in time, which by design will span three dimensions as well as fluid and kinetic scales

For the next few slides, we will be following the procedures laid out in [Pecora et al 2023 ApJL](#) and applied to MMS data in [Pecora et al 2023 PRL](#)

Lag-Baseline Log-Projection Diagnostic





The Swarm is the Observatory



We can calculate the Yaglom flux for each baseline pair in lag space

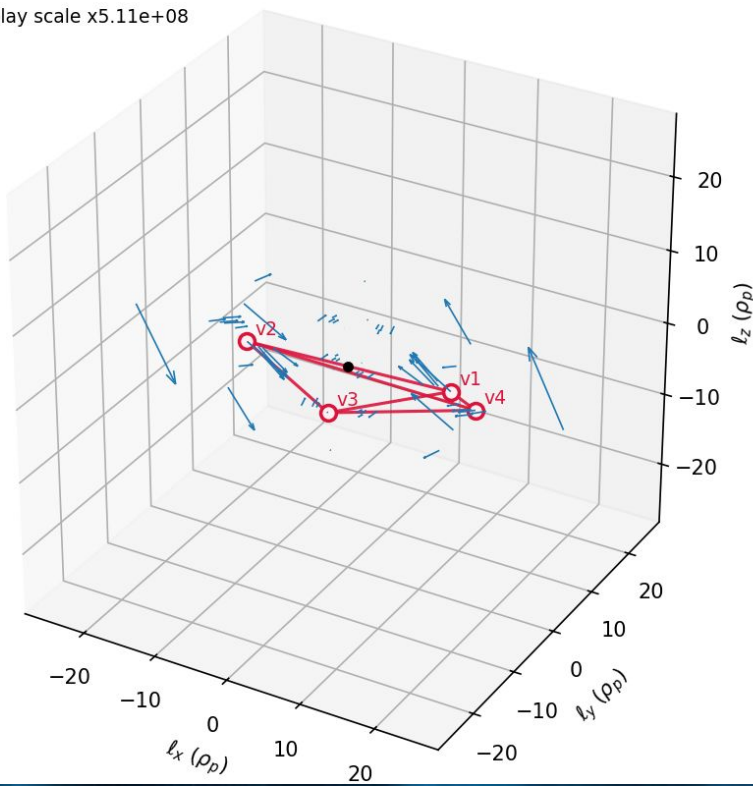
Lag-Space Time-Averaged Yaglom Flux

$$\langle \delta \mathbf{z}^\mp | \delta \mathbf{z}^\pm |^2 \rangle = Y^\pm$$

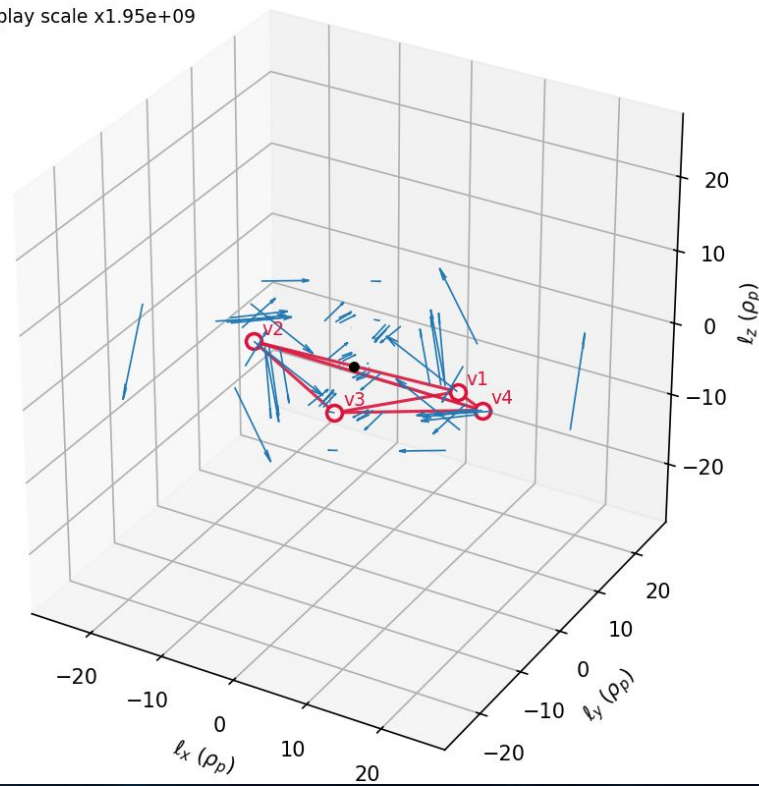
$\langle Y^+ \rangle$

$\langle Y^- \rangle$

Display scale x5.11e+08



Display scale x1.95e+09





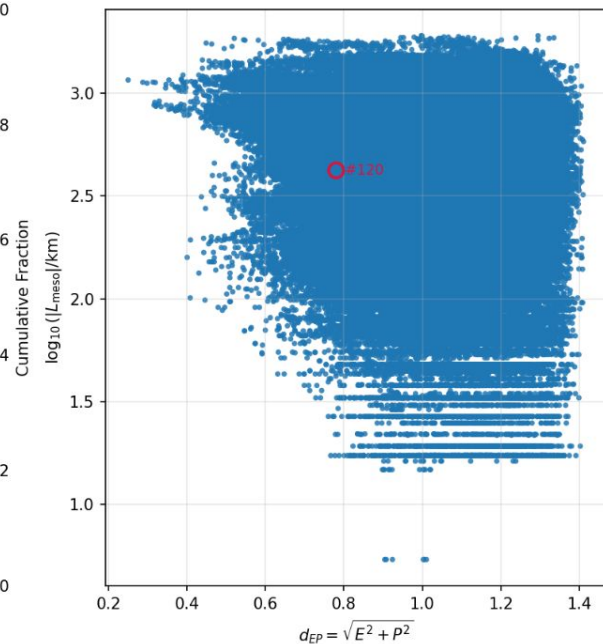
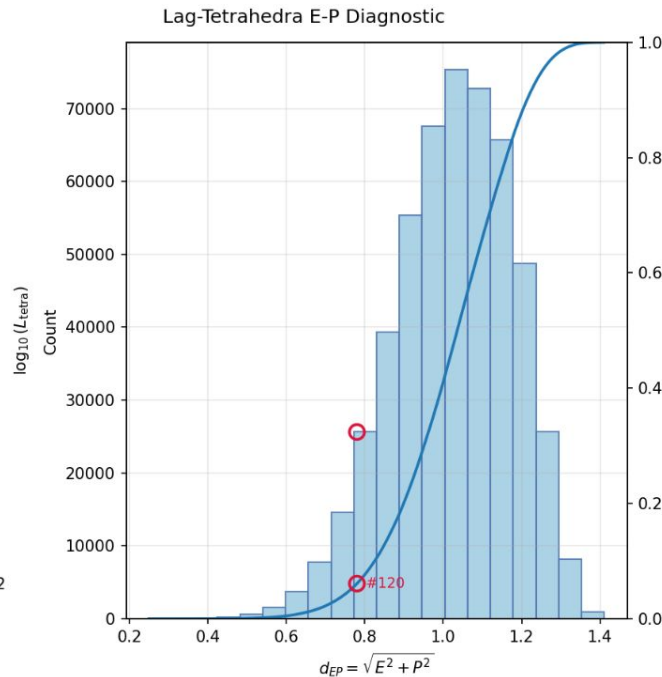
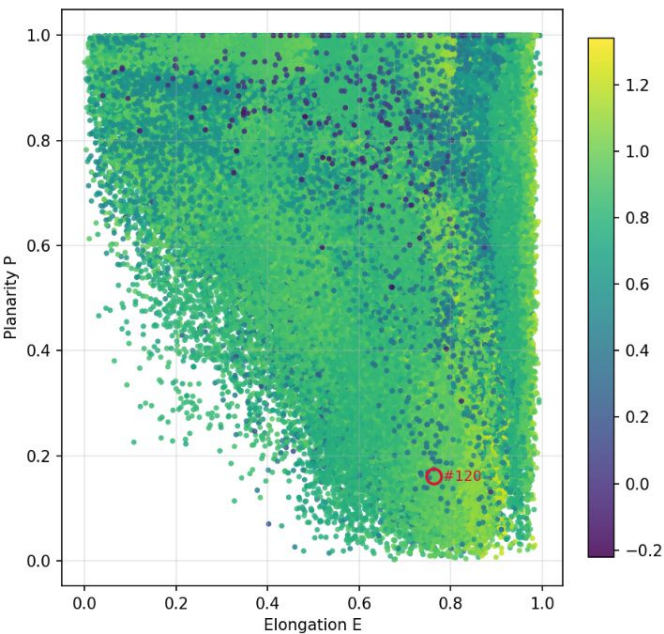
The Swarm is the Observatory



$$\nabla_{\ell} \cdot \mathbf{Y}^{\pm}$$

$C(36,4)=58,905$ Lag Tetrahedra

To evaluate Yaglom's law, we will need to calculate gradients in lag-space, using the same gradiometric techniques from real space (c.f.), just with more possible shapes.



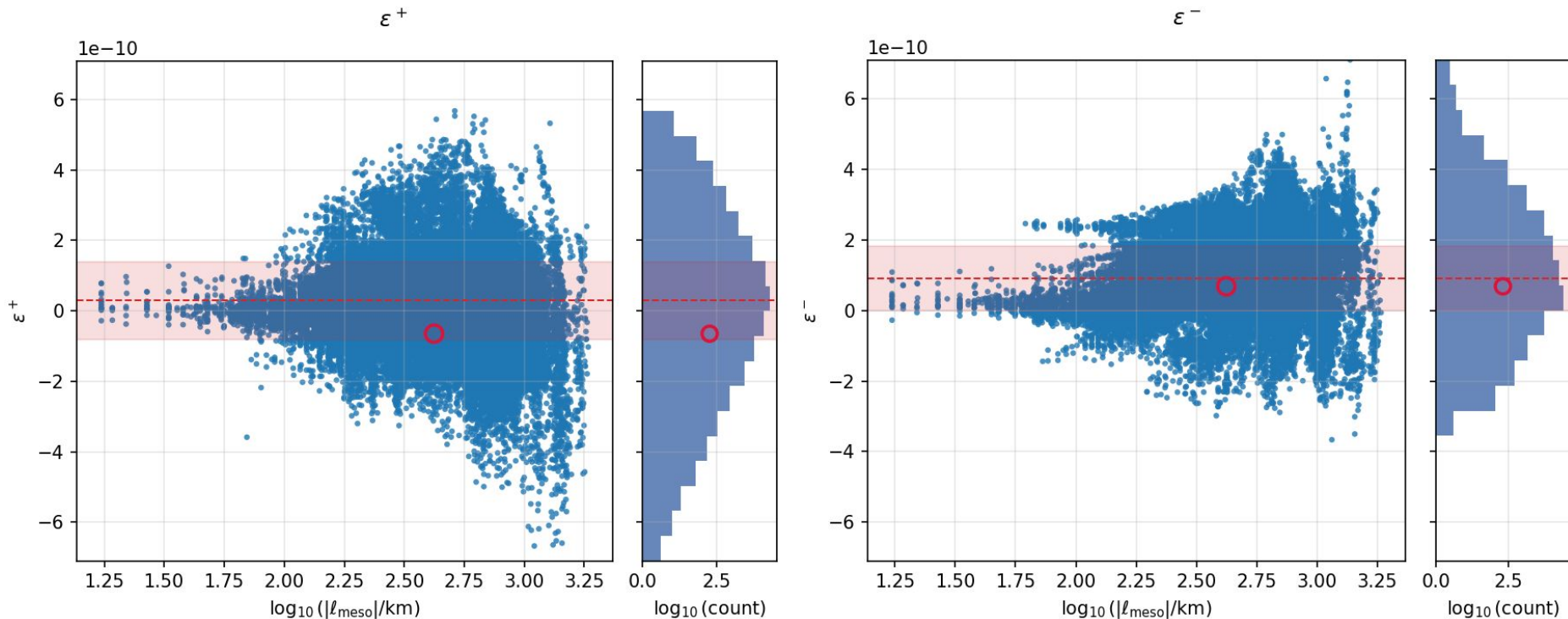


The Swarm is the Observatory

$$\nabla_{\ell} \cdot \mathbf{Y}^{\pm} = -4\epsilon^{\pm},$$

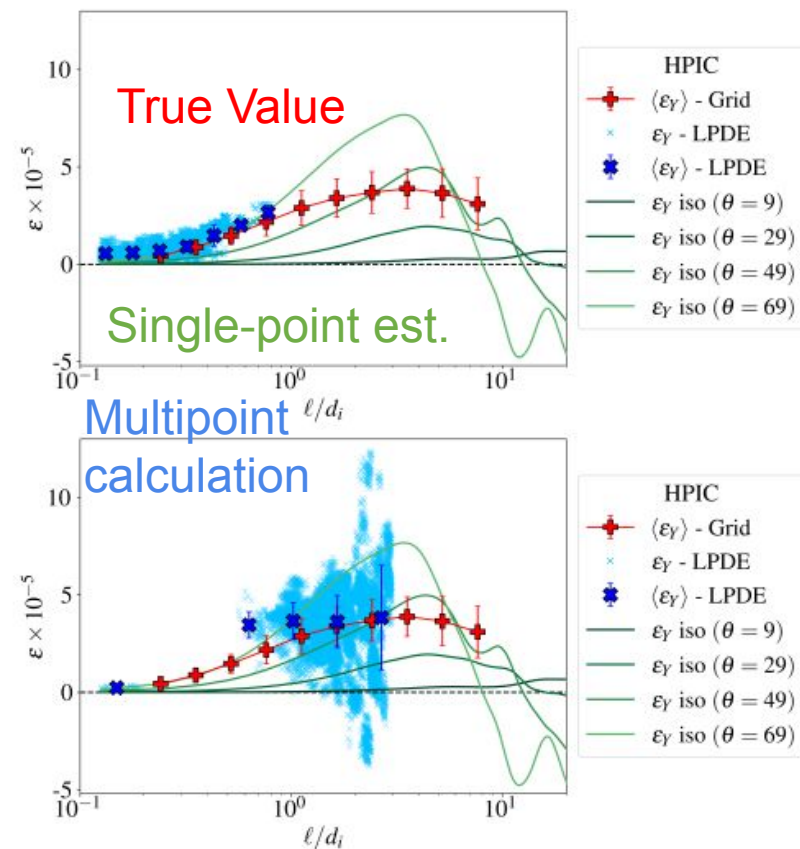
With the gradients determined, we have a statistical evaluation of the cascade rate as a function of scale.

Cascade-Rate Diagnostics





Impact of Yaglom's law anisotropy on the estimation of the turbulence energy transfer rate: A multipoint multiscale analysis Pecora et al 2026 A&A



We can compare the cascade rate can be extracted from the divergence of the Yaglom flux: $\nabla_l \cdot \langle \delta \mathbf{z}^\mp | \delta \mathbf{z}^\pm |^2 \rangle = 4\epsilon^\pm$

Against single point measurement that assume isotropy of the underlying fluctuations: which is sensitive to lags along V_{sw} :

$$3 \langle \delta \mathbf{z}^\mp | \delta \mathbf{z}^\pm |^2 \rangle / l = 4\epsilon^\pm \quad l = V_{sw} \tau$$

As magnetized turbulence is spectrally anisotropic, the impact of these assumptions must be interrogated.

[Pecora et al 2026](#) compares single point against multipoint methods developed for missions like HS (LPDE, [Pecora et al 2023 PRL](#)) and the known values extracted from numerical simulations.

Single-point methods depend on measurement direction, showing large variation with angle, with no unique answer. The errors from the assumption of isotropy persist across scales.

Multipoint method (Lag Polyhedral Derivative Estimate) recovers the **'true'** simulated energy transfer:

Measuring turbulence correctly requires simultaneous spatial sampling in multiple directions and scales. HS enables this for the first time, removing directional bias and accurately capturing energy transfer.



(G1) How is Turbulence Structured? How does it Evolve?

- G1O1: What is the (anisotropic) cascade rate?
- G1O2: How does the cascade rate change with system parameters?
- G1O3: How does energy move from flows to hotter plasma?
- G1O4: Do current sheets interrupt the cascade? What role does dynamic alignment play?

(G2) How do Boundaries and Structures Reshape Turbulence?

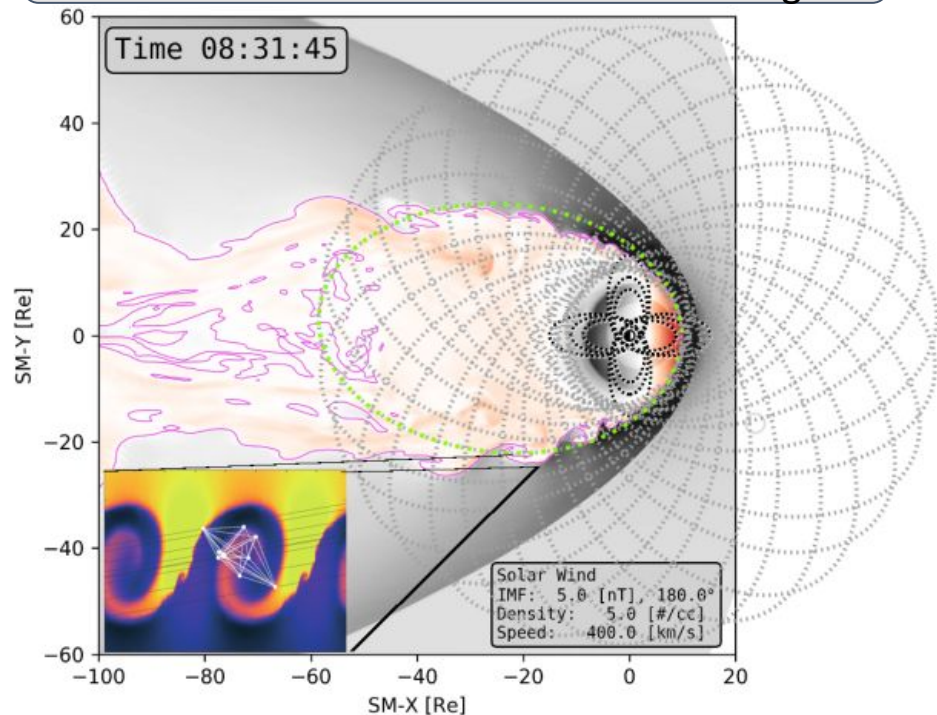
- G2O1: Do eruptions from the Sun restructure days-old turbulence?
- G2O2: How does turbulent structures change as it passes through the magnetosphere?



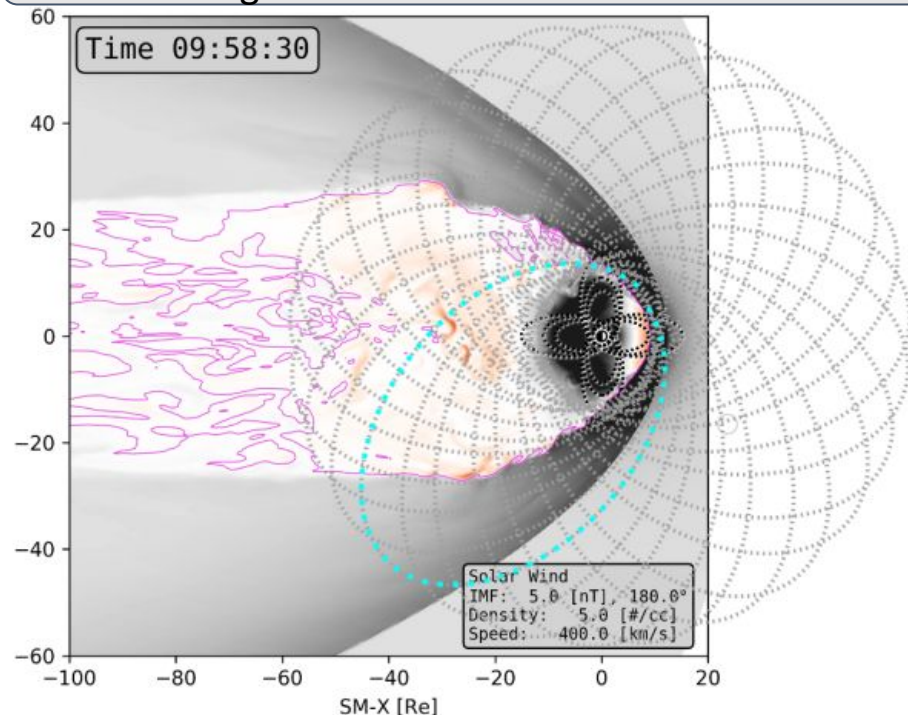
HS's Orbit Samples Key Magnetospheric Processes



HS measures the spatial structure of Kelvin Helmholtz wave formation and breaking



HS measures bursty bulk flows as they propagate from the magnetotail towards the Earth.



HS orbits (grey) superimposed on a simulations of the terrestrial magnetosphere.

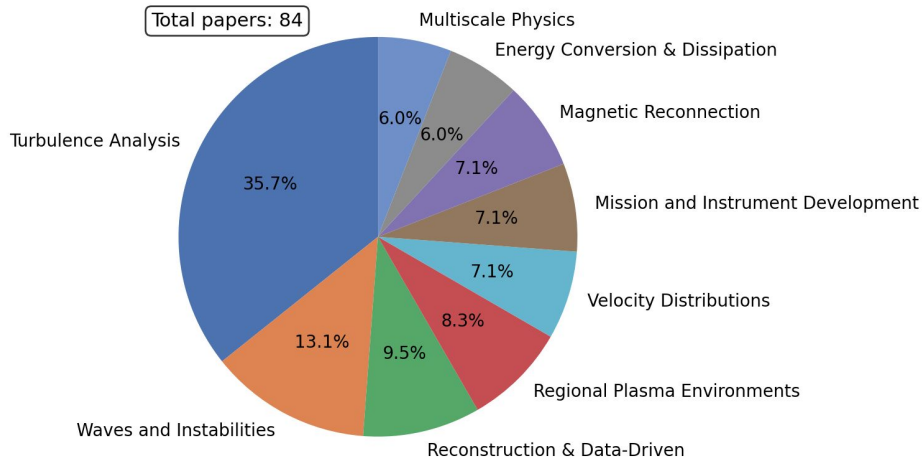
Sorathia et al 2020,
Archer et al 2024



HelioSwarm Science Extends Beyond Turbulence

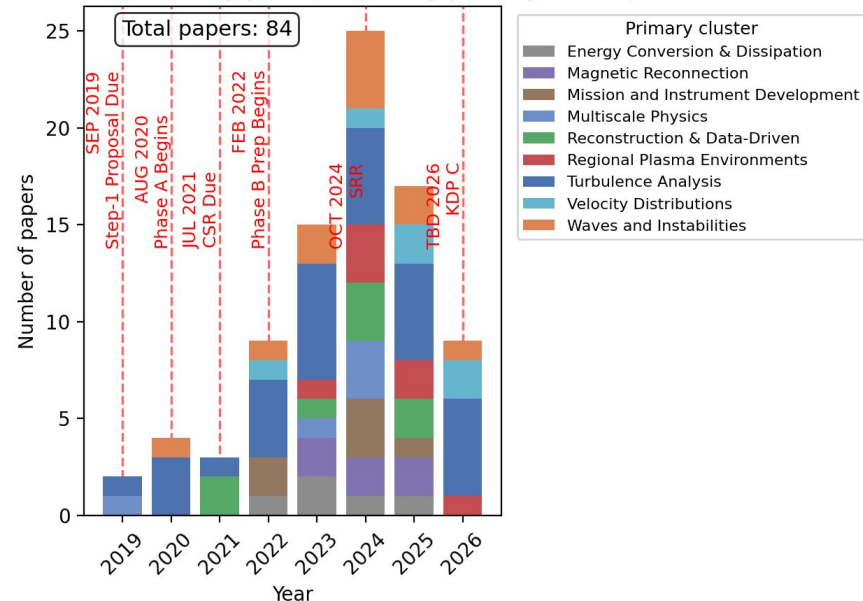


HelioSwarm literature by primary cluster



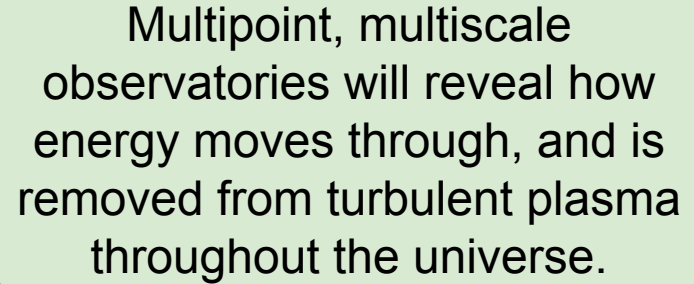
HelioSwarm-referencing publications span a broad range of topics, including both the prime science, motivation for future mission concepts, and bonus science enabled by the planned mission architecture.

HelioSwarm literature by year (stacked by primary cluster)



Papers Referencing the HS Mission can be found [here](#).

The logo for HELIO SWARM features the text "HELIO" in a large, bold, black sans-serif font, with "SWARM" in a slightly smaller, bold, black sans-serif font directly below it. To the right of the text is a black hexagonal icon containing a white geometric pattern of dots and lines, resembling a network or a stylized swarm. The background of the slide is a dark, textured image of a swarm of bees.



HS Trajectories



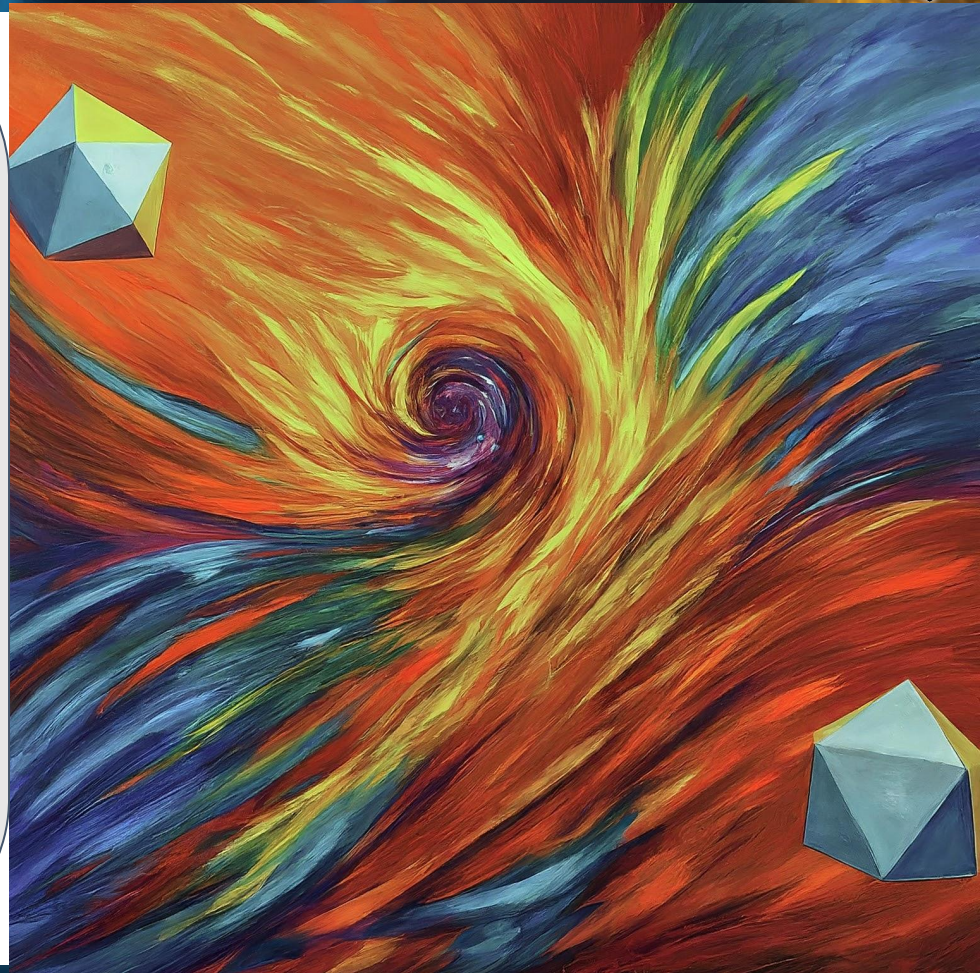
Backup Slides

Big plasmas have little plasmas,
That dance in magnetic fields;
And little plasmas have ions small,
Electric charges, they heed the call,

But in the plasma's complex space,
Anisotropy has its own place,
For magnetic fields can vary in direction,
Causing turbulence with different
perfection.

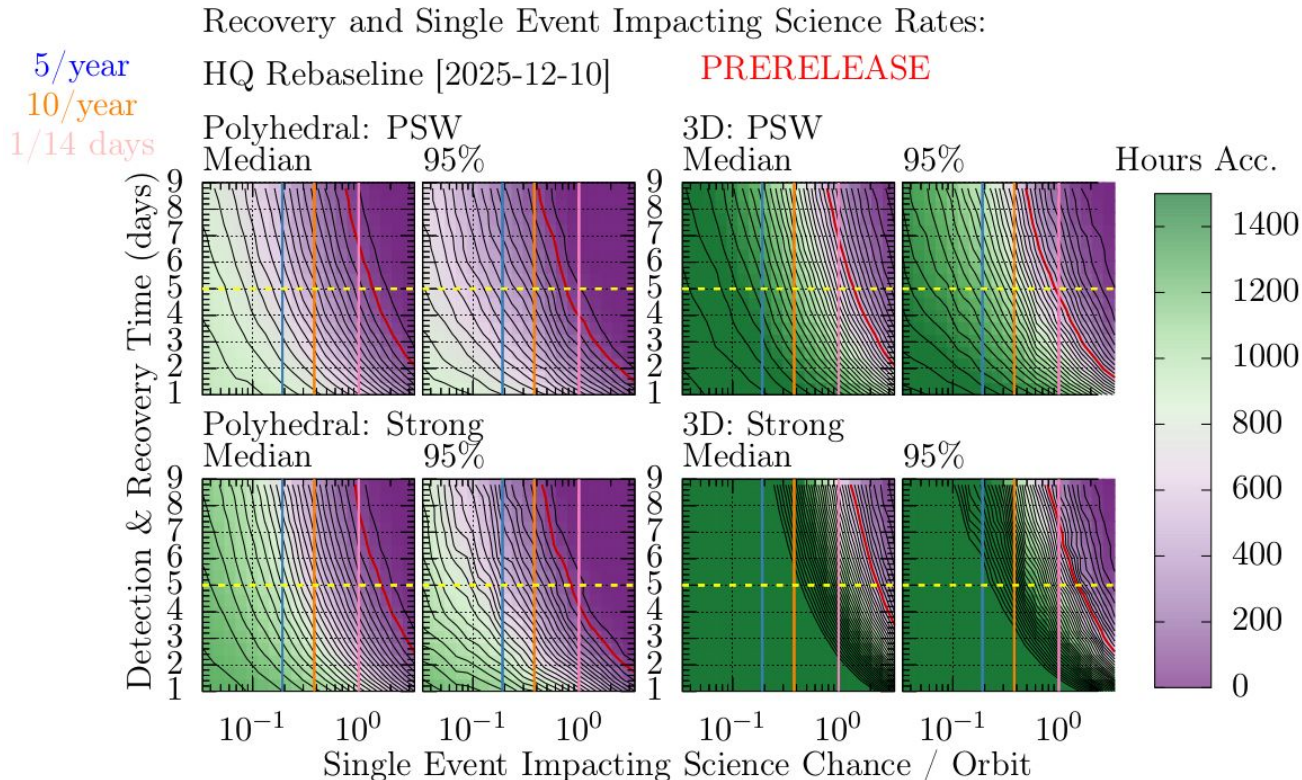
And intermittency too, plays a role,
In the plasma's dynamic soul,
For bursts of energy can come and go,
With changing frequencies and flow.

-Apologies to Richardson



To discern the impact of single events interruptions impacting science collection, we perform a Monte Carlo analysis, where each spacecraft is assigned a probability per hour of entering safe mode as well as a time required to detect and recover from these events.

Given these two rates, we determine the number of hours accumulated using an ensemble of 500 instantiations, and illustrate both the median and 95% element from the ensemble. The STL conservative estimate is the orange vertical line, and the red line is the associated required number of hours.

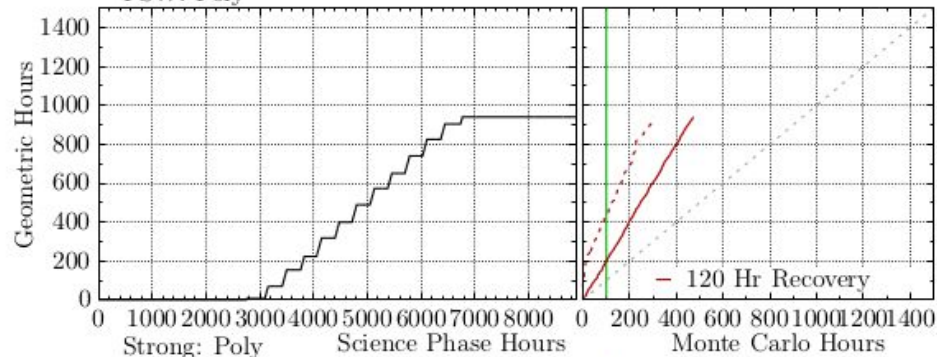


The observatory design satisfies the required number of hours with margin.

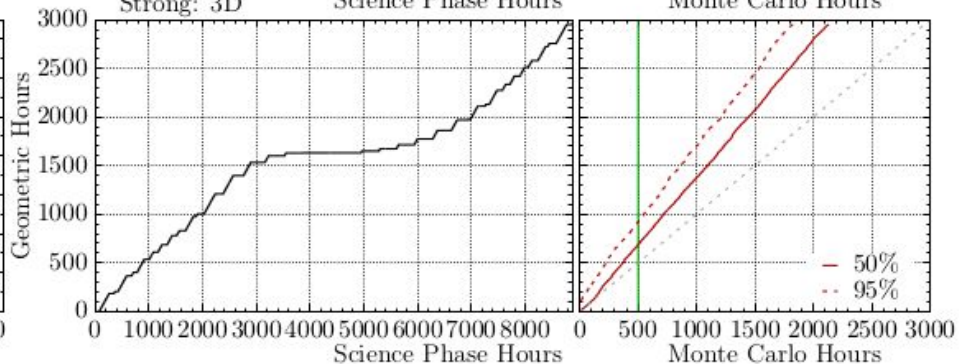
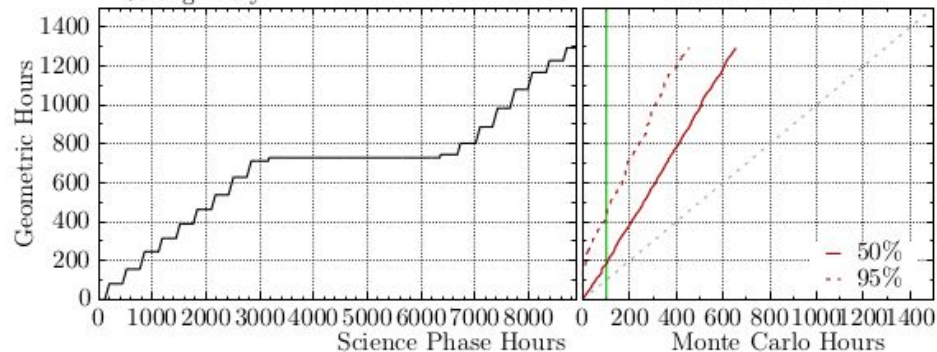
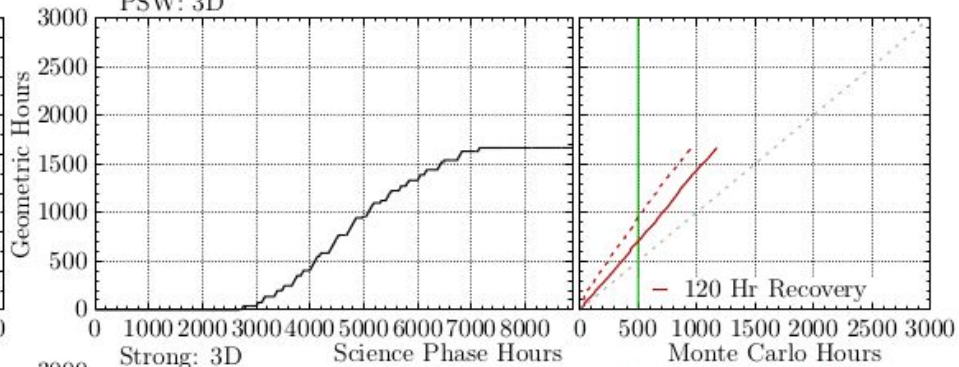
Focusing a 10x/year/node safe mode rate and a 5 day detect and recovery time, we can determine the expected number of hours necessary to accumulate the required hours (the green vertical line) at the 95% level (dashed red line).

The current design satisfies this confidence level for all four requirements.

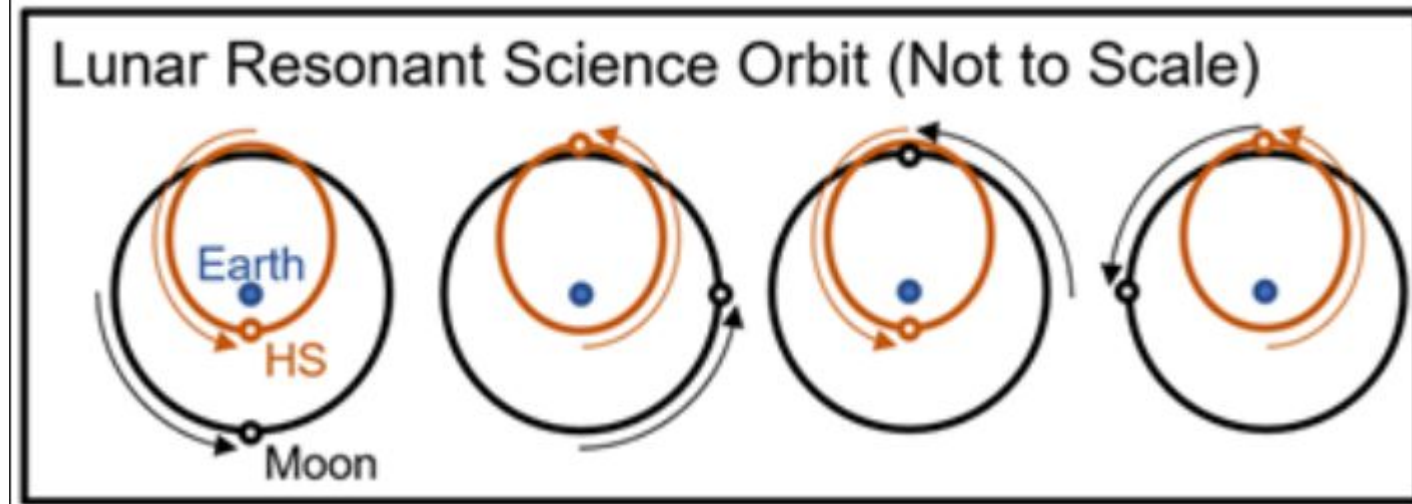
HQ Rebaseline: 0.384 Safe Mode/Node/Orbit
PSW: Poly



PRERELEASE
PSW: 3D



The Observatory (apart from the lunar swingby used to inject the flight segment into the science orbit) does not encounter the Moon, but keeps to a $P/2$ resonance.



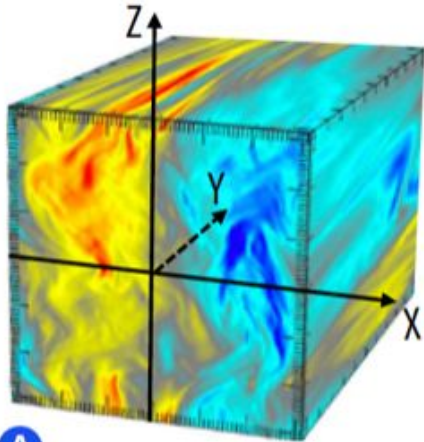


The Swarm is the Observatory



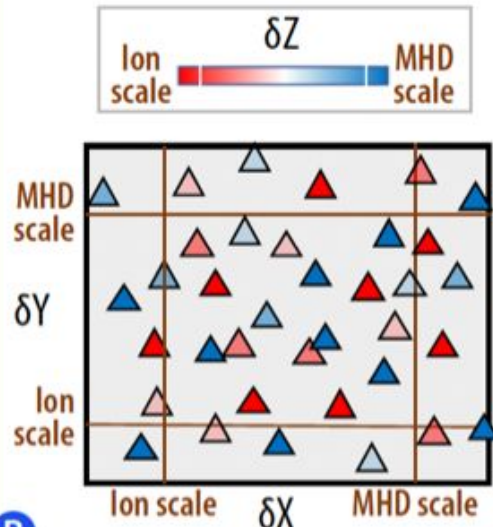
Solve:

Quantify energy cascade rate ($\epsilon^\pm$);
solve $\epsilon^\pm = -(1/4) \nabla \cdot \langle \delta \mathbf{z}^\mp | \delta \mathbf{z}^\pm |^2 \rangle$
using Elsasser fields ($\delta \mathbf{z}^\pm$) on 3D grid,
where $\delta \mathbf{z}^\pm = \mathbf{z}^\pm(\mathbf{x} + \delta \mathbf{x}) - \mathbf{z}^\pm(\mathbf{x})$
and where $\mathbf{z}^\pm = \delta \mathbf{v}_p \pm \delta \mathbf{B} / \sqrt{\eta_p}$



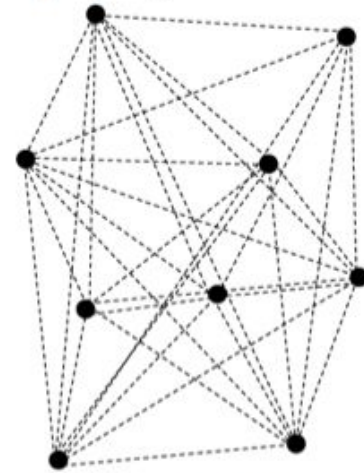
A

Approach: Create distribution of vector baseline separations ($\delta \mathbf{x}$) needed to estimate divergence of ensemble of measured Elsasser fields across characteristic turbulence scales



B

Implement: Measure $\eta_p(\mathbf{x}, t)$, $\delta \mathbf{v}_p(\mathbf{x}, t)$, $\delta \mathbf{B}(\mathbf{x}, t)$ at 9 locations ($j=1-9$); quantify $\mathbf{z}^\pm(\mathbf{x}, t)$, $\delta \mathbf{z}^\pm$, $\nabla \cdot \langle \dots \rangle$ using swarm as observatory



$\mathbf{x}_j(t)$: 3D location of s/c j at time t

$\delta \mathbf{x}_{jk}$: 36 baseline vectors between s/c j & k

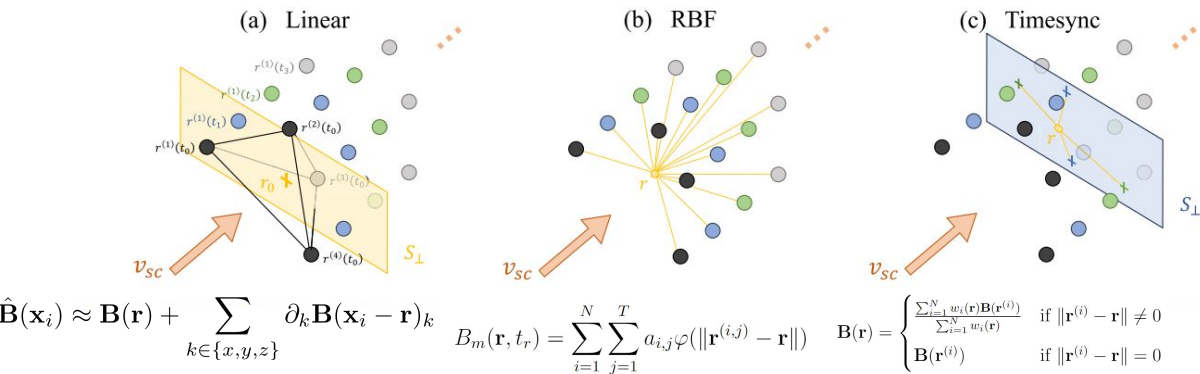
C

Measurements from all 28 instruments with separations spanning MHD and ion scales combine to enable novel characterizations of multiscale turbulent processes

Multi-Spacecraft Magnetic Field Reconstructions: A Cross-Scale Comparison of Methods: (Broeren et al. E&SS 2024)



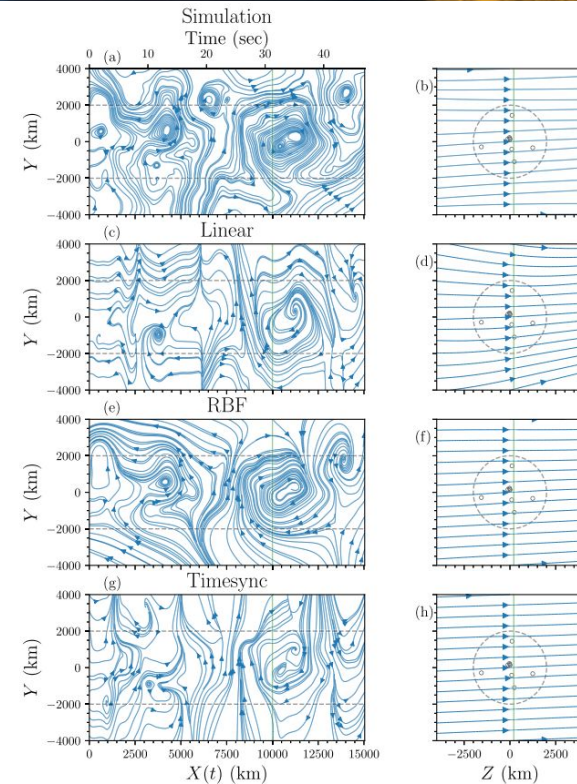
Reconstruction Methods



HelioSwarm enables the reconstruction of density, velocity, and magnetic fields from the ‘sparse’ set of nine timeseries using different methods.

The HS science team has been developing novel methods to leverage the high temporal cadence of the HS observations to more accurately reproduce a simulation of turbulence when sampling along representative HelioSwarm trajectories, when compared to typical linear gradient estimates.

While developing the methods, we also investigated a variety of error measures that prioritize different features of the turbulent field, e.g. the higher-order statistics or placement of large-scale structures.





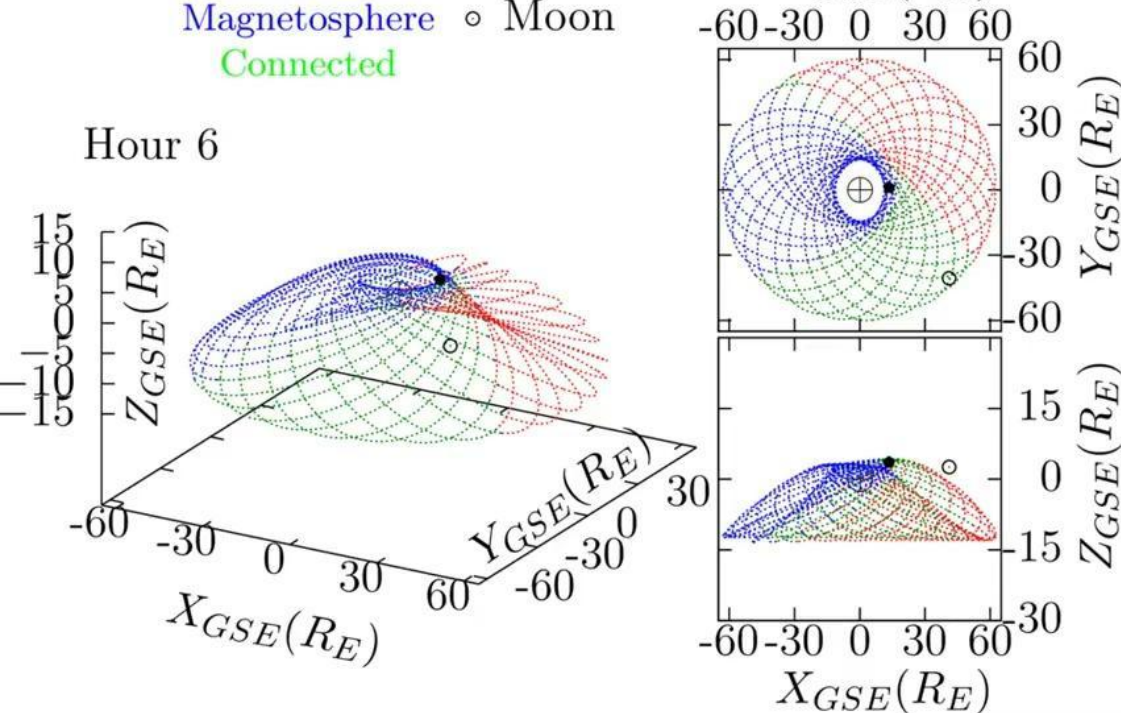
After reaching the science orbit, the nodes are released from the hub, after which the instruments undergo commissioning and the science phase begins.

The HelioSwarm Observatory: Phase-B Design Reference Mission

Solar Wind
Magnetosphere
Connected

• Observatory $X_{GSE}(R_E)$
○ Moon

Hour 6



HelioSwarm's L/2 orbit directly samples cislunar space, observing the **pristine solar wind**, parts of the **magnetosphere**, and the **magnetically connected foreshock**.

These different environments will enable tests of turbulence theories that depend on different driving and plasma conditions

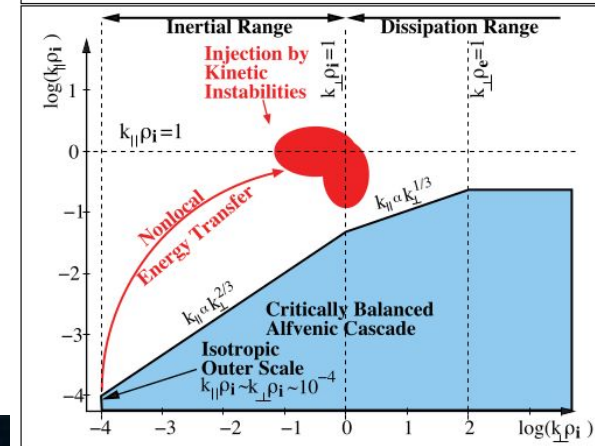
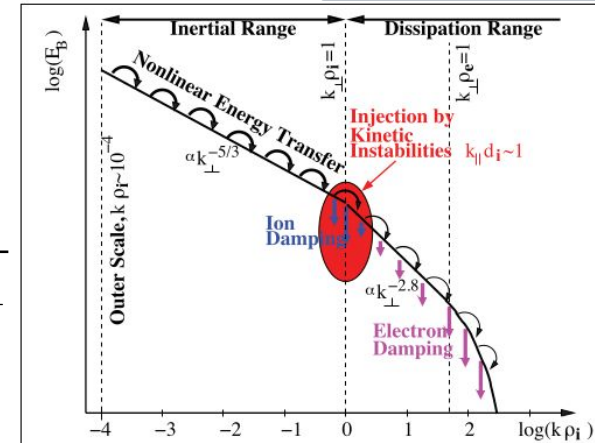
Trajectory
Videos are
available
here:



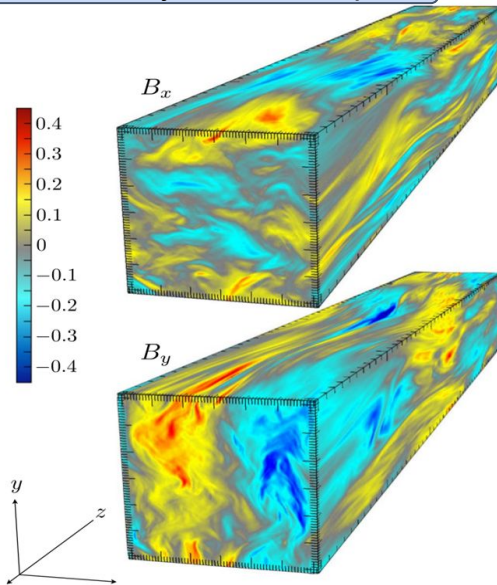
What Changes for Turbulent Plasmas?

Unlike in a fluid system, you can't Galilean transform away $\mathbf{B}$ as you can $\mathbf{u}$; the magnetic field leads to anisotropies in the transfer of mass and energy. Strong Plasma Turbulence settles into a balance between the linear propagation time and non-linear decorrelation time:

Howes 2024 JPP



Arzamasskiy et al 2019 ApJ



$$\tau_{\text{lin}} \sim \frac{l_{\parallel}}{v_{A0}} \quad \tau_{\text{nl}} \sim \frac{l_{\perp}}{\delta u_{\perp}} \sim \frac{l_{\perp}}{v_{A0}} \frac{B_0}{\delta B_{\perp}}$$

Leading to a preferential transfer of energy to smaller perpendicular rather than parallel scales.

The refinement of this anisotropic model of **critical balance** can be followed through Goldreich & Sridhar 1995, Boldyrev 2006, Mallet et al 2015, Schekochihin 2020...

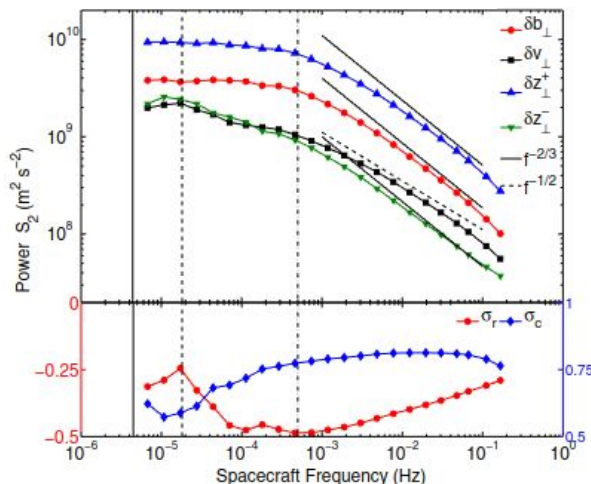
Quantifying the Plasma State

To quantify the plasma state at MHD and ion scales, we need to resolve the density n , the vector velocity $\mathbf{v}$, the magnetic field $\mathbf{B}$, and the tensor pressure $\mathbf{P}$.

These allow us to construct the Elsasser Variables $\mathbf{z}$, which describe the incompressible Alfvénic fluctuations:

$$\mathbf{z}^{\pm} = \mathbf{v}_{\perp} \pm \frac{\delta \mathbf{B}_{\perp}}{\sqrt{\mu_0 \rho_0}}.$$

$$\frac{\partial \mathbf{z}^{\pm}}{\partial t} \mp (\mathbf{v}_A \cdot \nabla) \mathbf{z}^{\pm} + (\mathbf{z}^{\mp} \cdot \nabla) \mathbf{z}^{\pm} = -\nabla p$$



These parameters can be recombined to describe other aspects of the turbulence, e.g. the cross helicity σ_c , which describes the imbalance in the power moving along vs. against the mean magnetic field direction.

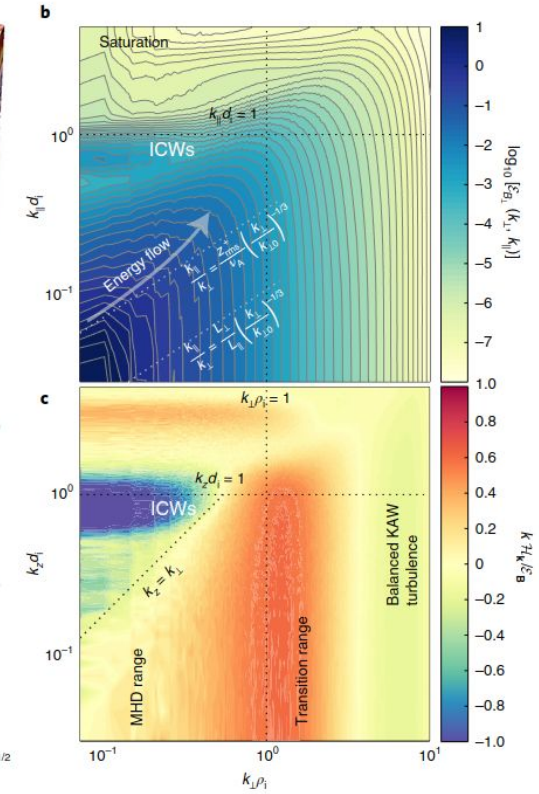
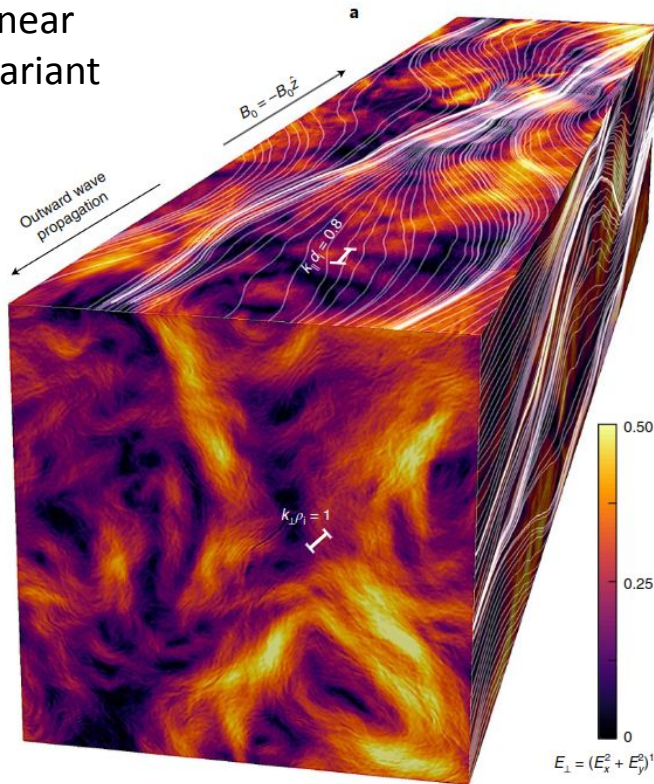
$$\begin{aligned} \sigma_c(t, \tau) &= \frac{2\delta \mathbf{v}_{\perp}(t, \tau) \cdot \delta \mathbf{b}_{\perp}(t, \tau)}{|\delta \mathbf{v}_{\perp}(t, \tau)|^2 + |\delta \mathbf{b}_{\perp}(t, \tau)|^2} \\ &= \frac{|\delta \mathbf{z}_{\perp}^{+}(t, \tau)|^2 - |\delta \mathbf{z}_{\perp}^{-}(t, \tau)|^2}{|\delta \mathbf{z}_{\perp}^{+}(t, \tau)|^2 + |\delta \mathbf{z}_{\perp}^{-}(t, \tau)|^2}, \end{aligned}$$

Helicity Barrier: A New Model for Imbalanced Systems

In imbalanced Alfvénic turbulence, nonlinear conservation of a generalized helicity invariant creates a “barrier” at the ion-gyroscale.

Above a critical electron plasma β , the cascade of helicity cannot keep up with that of free energy, so the turbulent transfer of energy to sub-ion scales is halted, enabling the creation of parallel structures with finite size. These structures dump energy into the ions, rather than continuing to electron scales.

While PSP has enable some statistical observational tests of this theory (McIntyre in prep, Bowen et al 2024 Nat Astro.) these analysis have been performed at single points in space.



See Meyrand et al 2021 JPP and Squire et al 2022 Nat Astro. for a detailed derivation.

Recovering Ion Distribution Functions: I. Slepian Reconstruction of VDFs from MMS and Solar Orbiter (Das and Terres 2025)

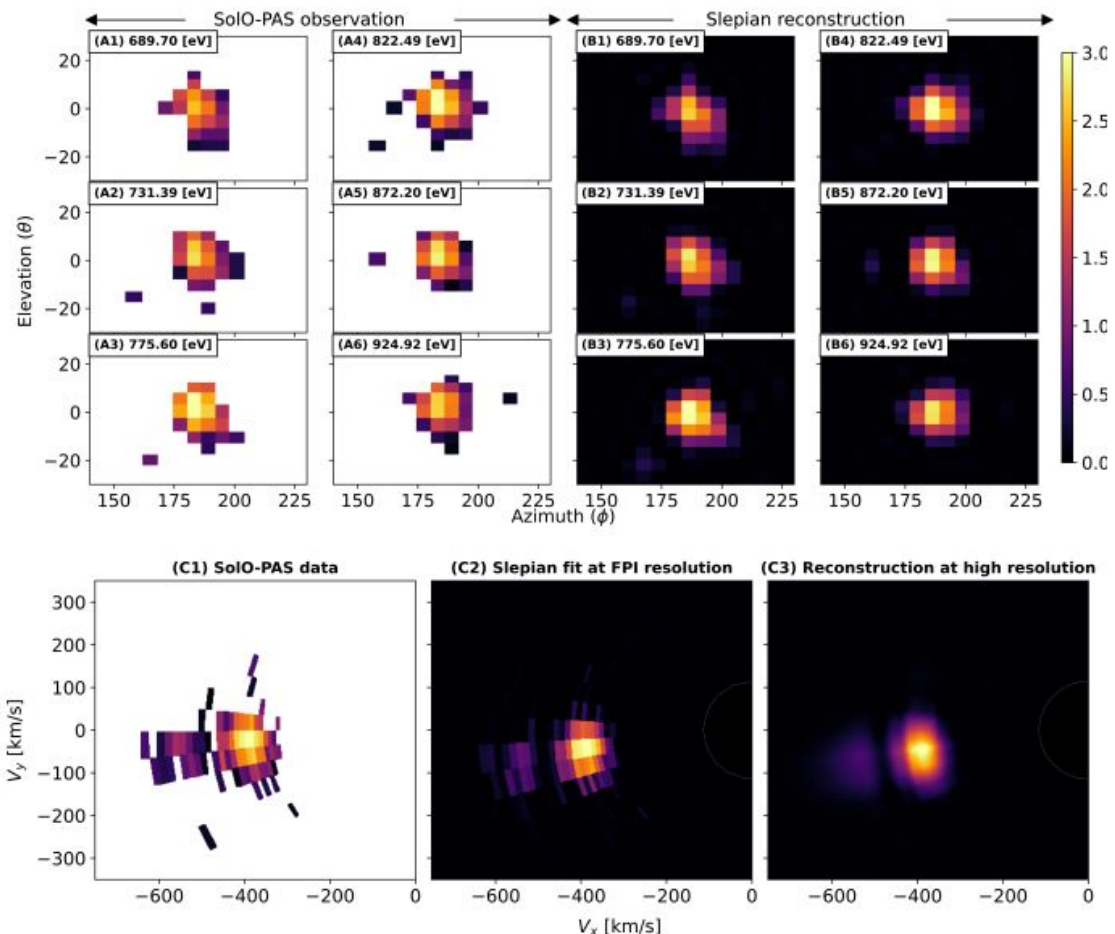
Ion Velocity Distributions will be measured in detail with the iESA on a fixed grid in (E, ϕ, θ) .

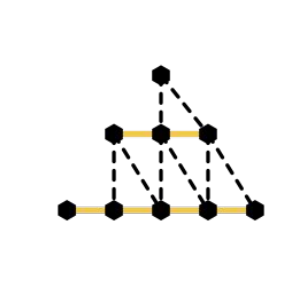
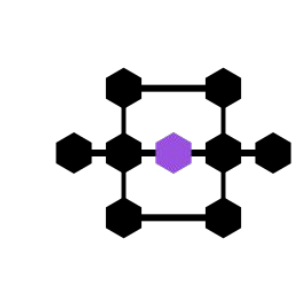
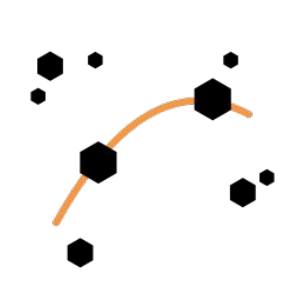
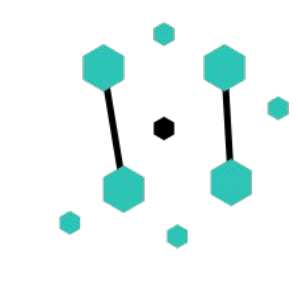
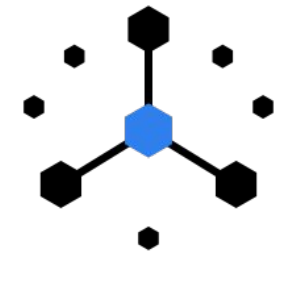
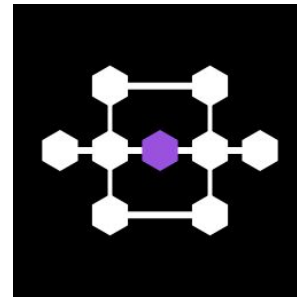
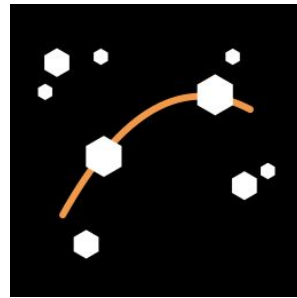
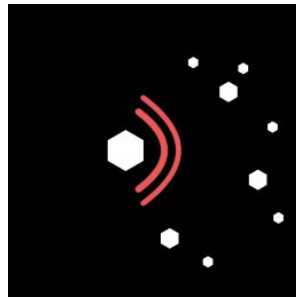
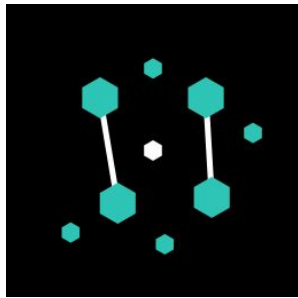
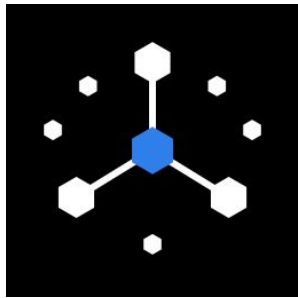
Extracting insights into the underlying kinetic plasma processes *typically* has required fitting the VDF as a one or several biMaxwellians. This leaves out many of the interesting bumps and wiggles that are signatures of various instabilities and damping processes.

Das and Terres have implemented an improved fits using a spatospectrally concentrated set of basis functions (the Slepian) that has been used in other areas of geophysics. It preserve the VDF structure, as well as requiring fewer functions than globally defined basis (e.g. spherical harmonics). They have implemented this on SoLO PAS, the instrument that provides heritage for HS's iESA.

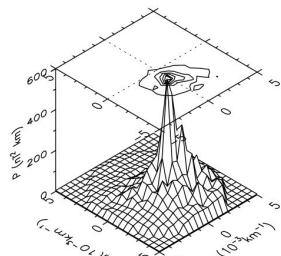
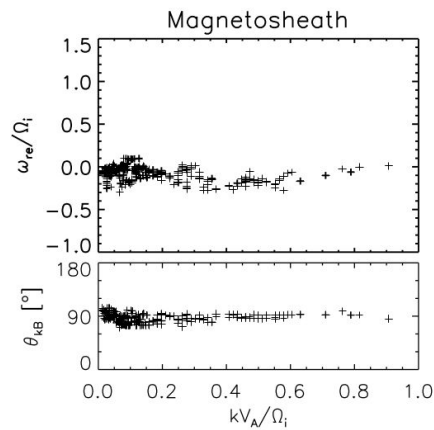
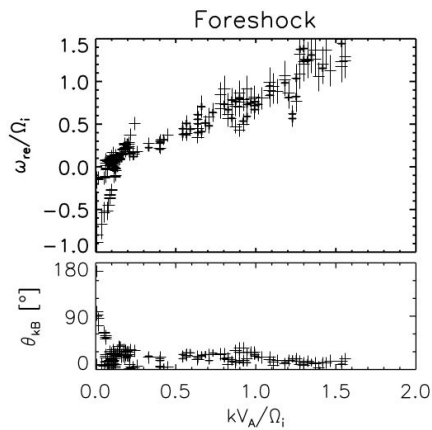
"[This] framework of applying the SBR method to MMS and SoLO data has been developed under a Python and Matlab based package, which we call `vdFit`. Future work will extend this framework to include solar wind ESA instruments PSP-SPAN and HelioSwarm"

<https://arxiv.org/pdf/2501.17294>





Spatial Gradients Require Polyhedra



Narita et al 2022 JGR

In order to measure changes in three dimensional structure and gradients requires measurements of the plasma and magnetic field made at at least four space (i.e. the vertices of a **tetrahedra** or more general **polyhedra**). The canonical example is the **curlometer** method, which enables the measurement of the current from the curl of the magnetic field.

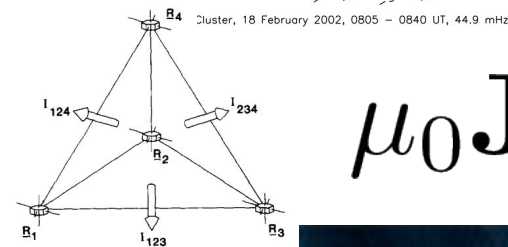
These can be coupled with measurements of the temperature at a single point to quantify the plasma heating.

Wave telescope: Multipoint method for determining the wavevectors of plasma waves and their associated 3D power distributions.

Pressure-strain interaction (Pi-D): Measurement of the dilation, $-(\mathbf{P} \cdot \nabla) \cdot \mathbf{v}$, describing the local conversion between flow and thermal energy.

Curlometer & Gradient Methods: Multipoint methods for constructing current density/intermittent structures from spatially distributed measurements.

$$\mu_0 \mathbf{J} = \nabla \times \mathbf{B}$$

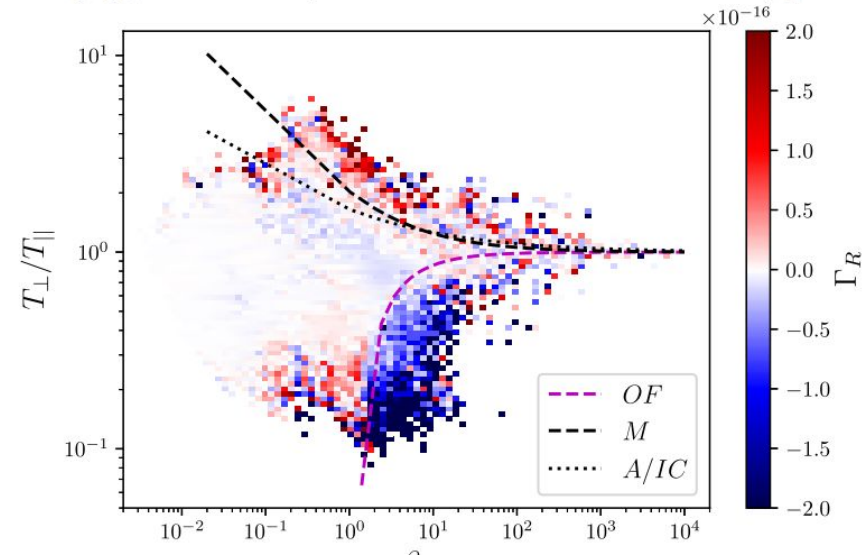


Temperature anisotropy instabilities driven by intermittent velocity shears in the solar wind (Opie 2024 JPP)

- Identifies the driving of instabilities with a “radial rate of strain” (Γ_R), effectively quantify how velocity shears drive temperature anisotropy in the solar wind.
- Using measurements from Solar Orbiter, they demonstrate that unstable plasma intervals exhibit significantly higher levels of shear compared to stable intervals, providing quantitative hints between turbulent structures and the generation of unstable waves.
- **This method only captures a single direction of the strain; to fully capture the tensor, multiple spacecraft are needed.**

$$\frac{d}{dt}(p_{\perp} - p_{\parallel}) = (p_{\perp} + 2p_{\parallel})\hat{b}\hat{b} : \nabla \mathbf{v} - (2p_{\perp} - p_{\parallel})\nabla \cdot \mathbf{v}$$

$$\Gamma_R = \frac{1}{\tau} \left\{ \left[(p_{\perp} + 2p_{\parallel}) \left(\hat{b}_R \hat{b}_R \Delta v_R + \hat{b}_T \hat{b}_R \Delta v_T + \hat{b}_N \hat{b}_R \Delta v_N \right) \right] - (2p_{\perp} - p_{\parallel}) \Delta v_R \right\}$$



“Our work highlights new science opportunities for multi-spacecraft missions such as **HelioSwarm**...which will provide a means of measuring fully three-dimensional rates of strain and intermittency. These data will enable a more precise unpicking of the contributions of non-compressive and compressive fluctuations as well as intermittency to the creation and modulation of temperature anisotropy. A mission like HSwi with their complements of fields and plasma instrumentation has thus the potential to create breakthroughs in our understanding of the interplay between turbulence and kinetic instabilities in space plasmas. In particular, our results suggest that accurate and high-resolution measurements of the velocity field will be of fundamental importance to this task.” <https://ui.adsabs.harvard.edu/abs/2024JPIPh..90f9002O/abstract>

Evaluating the fine structure of magnetic field topologies requires estimating both linear and quadratic gradients, e.g. the Taylor Expansion

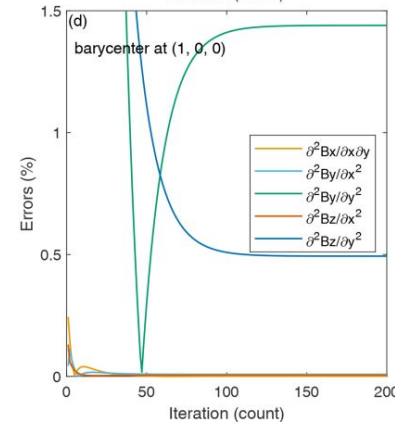
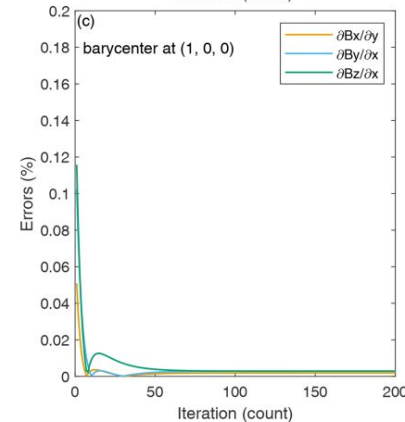
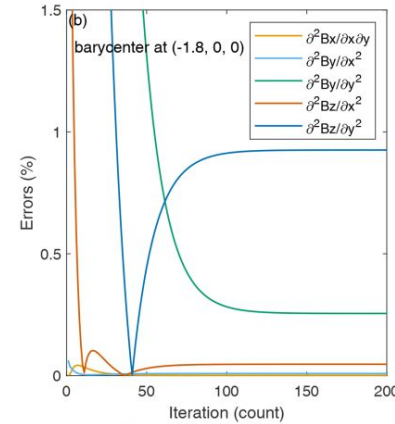
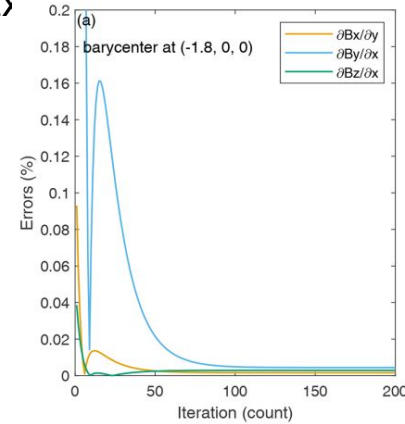
$$\mathbf{B}(t, \mathbf{r}) = \mathbf{B}(t, \mathbf{r}_c) + (\mathbf{r} - \mathbf{r}_c) \cdot \nabla \mathbf{B}(t, \mathbf{r}_c) + \frac{1}{2}(\mathbf{r} - \mathbf{r}_c)(\mathbf{r} - \mathbf{r}_c) \cdot \nabla \nabla \mathbf{B}(t, \mathbf{r}_c).$$

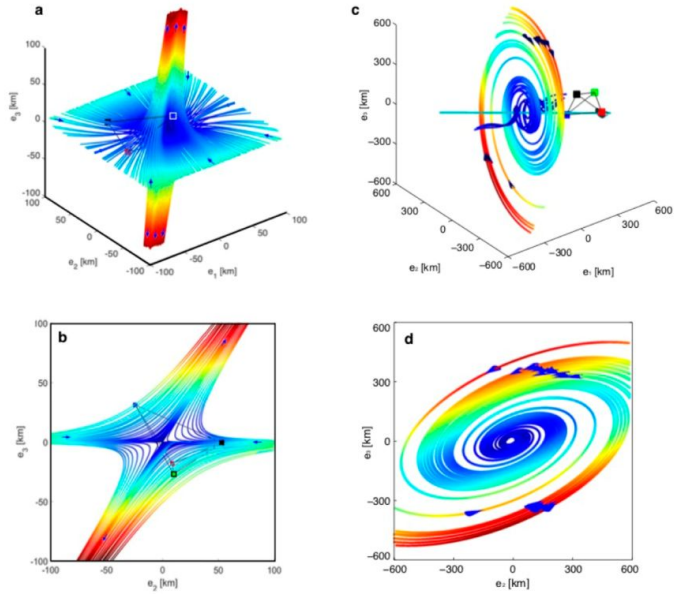
Traditionally (Chanteur 1998), there are ten unknown parameters, requiring 10 measurement points to resolve.

Shen et al 2025 have developed a novel method that combines a mixed temporal and spatial derivatives

$$\partial_t \mathbf{B} = -\mathbf{V} \cdot \nabla \mathbf{B} \text{ and } \partial_t \nabla \mathbf{B} = -\mathbf{V} \cdot \nabla \nabla \mathbf{B}.$$

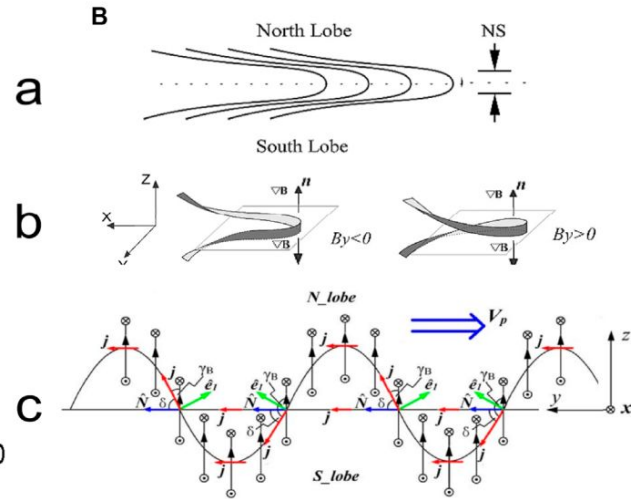
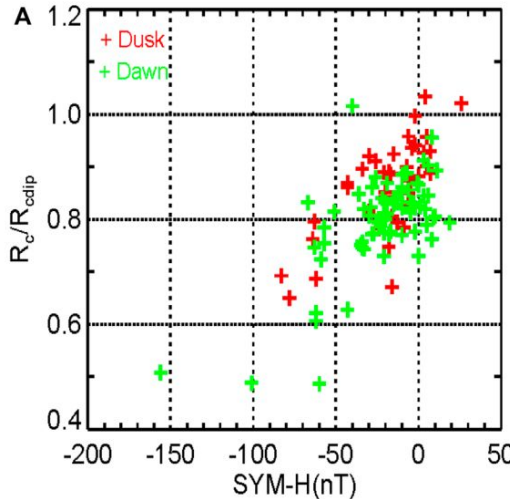
with an interactive least-squares approach that enables the resolution of both the linear and quadratic terms with only 7 S/C.





A review of the development and application of multi-spacecraft analysis methods, emphasizing the **curlometer technique** for calculating current from magnetic field measurements. Originally developed for the Cluster mission, these methods have been adapted for other missions like MMS, THEMIS, and Swarm.

The paper also highlights advancements in spatial gradient and magnetic topology analyses, including methodologies like Taylor expansion (FOTE), least-squares approaches, and integral theorems. These techniques enable detailed investigations of magnetic field gradients, and field-aligned phenomena across diverse spatial scales and spacecraft configurations.



This paper advocates for further refinement of methodologies to leverage larger spacecraft arrays and discusses their potential for upcoming missions like HelioSwarm.

It can be shown (Politano et al 1998) that the cascade rate can be quantified using a function of increments of the Elsasser variables $\mathbf{z}^{\pm} = \mathbf{v} \pm \mathbf{B}/\sqrt{4\pi n}$ $\delta \mathbf{z}^{\pm}(\mathbf{x}, \mathbf{l}) = \mathbf{z}^{\pm}(\mathbf{x} + \mathbf{l}) - \mathbf{z}^{\pm}(\mathbf{x})$

This function is known as the Yaglom Flux: $\mathbf{Y}^{\pm} = \langle \delta \mathbf{z}^{\mp} | \delta \mathbf{z}^{\pm} |^2 \rangle$

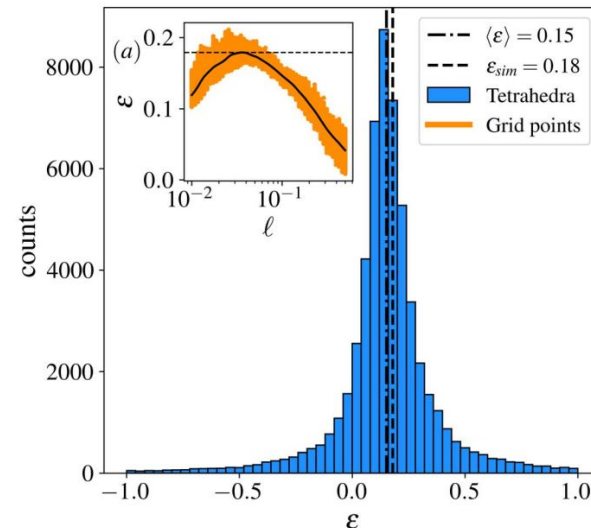
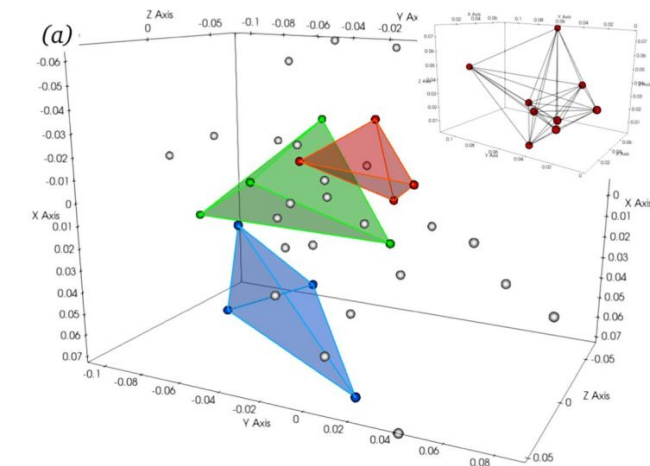
And within the inertial range, its divergence (in lag space) is related to the cascade rate: $\nabla \cdot \mathbf{Y}^{\pm} = -4\epsilon^{\pm}$

Comparisons with synthetic HS-like data compares well rates extracted from turbulence simulation.

With 9 spacecraft, we have 36 independent measurements of increments.

This enables us to build $C(36,4)=58905$ tetrahedra in lag space, which can be used to quantify the changes in the Yaglom flux as a function of scale, determining the cascade rate.

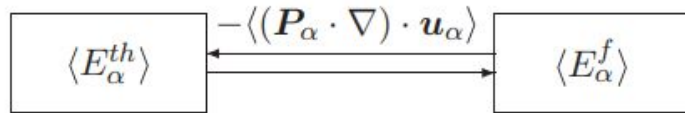
THE ASTROPHYSICAL JOURNAL LETTERS, 945:L20 (7pp), 2023 March 10



One of the multipoint analysis approaches that will be used by HS is the pressure-strain methodology, which measures energy conversion via the pressure strain term

$$(\mathbf{P}_p \cdot \nabla) \cdot \mathbf{V}_p$$

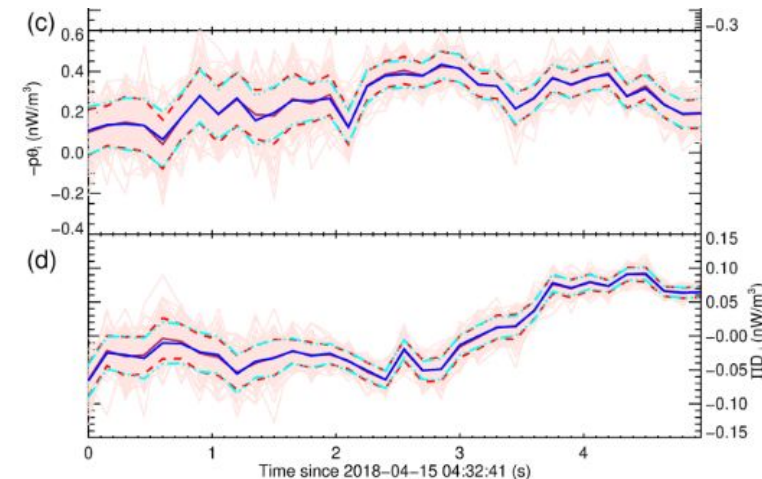
which quantifies the conversion of energy between flow (E^f) and internal (or kinetic, E^{th}) reservoirs. ($\mathbf{P}$ is the pressure tensor, and $\mathbf{V}$ is the bulk velocity).



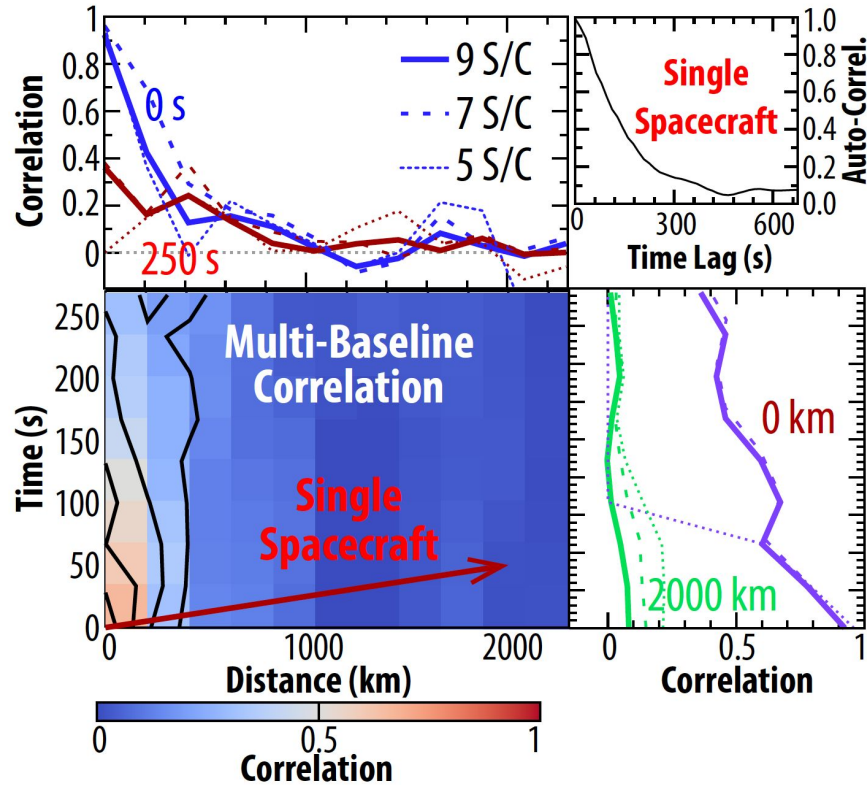
Yang et al 2017

The calculation of the spatial gradient requires at least four measurement points of $\mathbf{V}$; Roberts et al calculated the errors in all of the terms of this contraction using MMS observations.

Similar error analysis will be applied to HS observations.



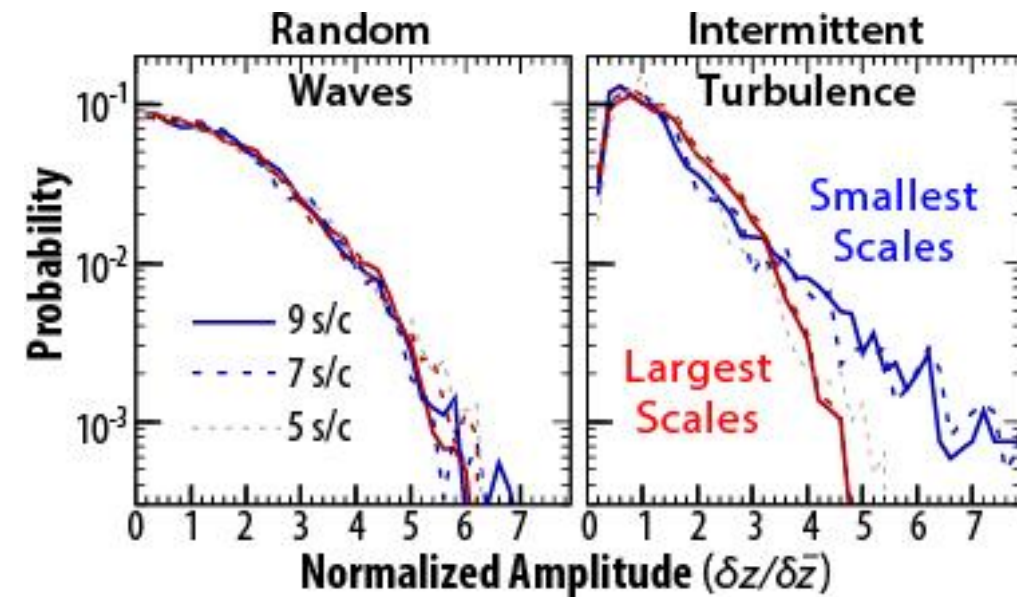
Analysis Approach: Time-Space Correlation



The autocorrelation of a time series measured by a single spacecraft with a lagged copy of said series fundamentally can not deconvolve the decorrelation of a turbulent signal as a function of time and space independently; see red arrow in lower left hand corner.

By performing two-point correlations between timeseries measured by all nine HelioSwarm spacecraft, we can separately measure the decorrelation times and length scales, accessing fundamental characteristics of the underlying turbulence.

Analysis Approach: More Structure Functions



The Nature of Turbulence in Space Plasmas

Calculating structure functions of measurements of plasma reveals variations in the statistical characteristics of the underlying turbulence, for instance how does the intermittent nature of the turbulence change for larger or smaller scale separations. This is typically done using an autocorrelation with a single time series.

HelioSwarm's 36 baseline separations between spacecraft will enable the calculation of such structure functions at both larger and smaller separations simultaneously, rather than needing to wait hours or days to collect sufficient data, potentially mixing different kinds of turbulence into a single calculation.

Shown is the calculation of the probability distribution function of the amplitude of the normalized Elsasser fluctuations for both large MHD and small ion kinetic scales (red and blue respectively, following Mallet et al 2015) drawn from HelioSwarm-like measurements of random waves, left, and a hybrid numerical simulation of turbulence, right.

Given a set of N-spatially distributed (Fourier Transformed) measurements of a field

Yields the distribution of power as a function of frequency and wavevector:

$$\mathbf{A}(\omega) = \begin{pmatrix} \mathbf{A}(\omega, \mathbf{r}_1) \\ \mathbf{A}(\omega, \mathbf{r}_2) \\ \vdots \\ \mathbf{A}(\omega, \mathbf{r}_N) \end{pmatrix}.$$

$$P(\omega, \mathbf{k}) = \text{Tr} \left\{ (\mathbf{H}^\dagger(\mathbf{k}) \mathbf{M}^{-1}(\omega) \mathbf{H}(\mathbf{k}))^{-1} \right\}.$$

With Four Spacecraft, this methods can be applied to accurately reconstruct power associated with several waves simultaneously, but only over a finite range of

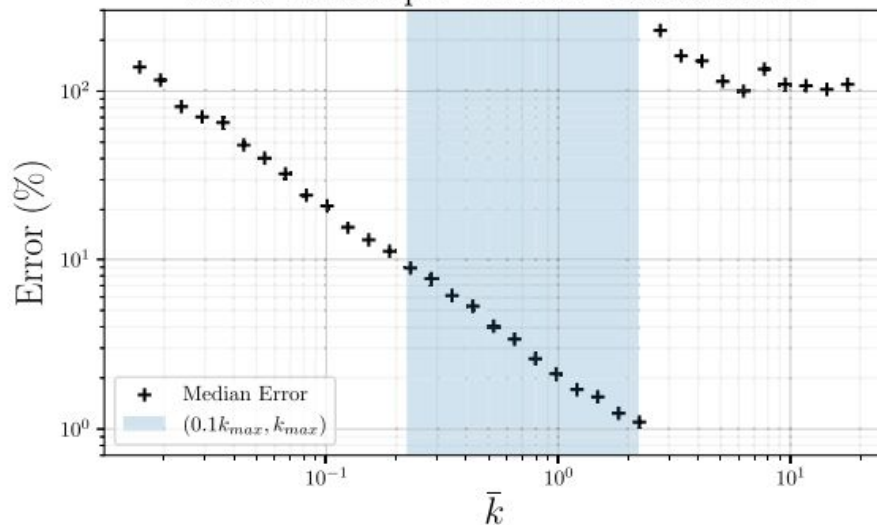
We can construct a spatial correlation matrix

$$\mathbf{M}(\omega) = \langle \mathbf{A}(\omega) \mathbf{A}^\dagger(\omega) \rangle.$$

Which combined with the steering vector

$$\mathbf{H}(\mathbf{k}) = \begin{pmatrix} \mathbf{I}_L e^{i\mathbf{k} \cdot \mathbf{r}_1} \\ \mathbf{I}_L e^{i\mathbf{k} \cdot \mathbf{r}_2} \\ \vdots \\ \mathbf{I}_L e^{i\mathbf{k} \cdot \mathbf{r}_N} \end{pmatrix},$$

Wave-Telescope: Perfect Tetrahedron





Extending the Wave Telescope

To optimally calculate the Wave-Telescope for N spacecraft, we determine the effective error for each N -spacecraft subset for any configuration. By selecting the subset at each k -magnitude that minimizes the value of $\mu_{\text{eff}}(\bar{k}, \chi) + 2\sigma(\bar{k}, \chi)$, we can increase the range of wavevectors over which we can accurately scan using the wave telescope technique.

HS Config: Hour 1

This expression allows the calculation of the effective error for each M -spacecraft subset for any configuration. By selecting the subset at each wavevector that minimizes the error, we can increase the range of wavevectors over which we can accurately scan using the wave telescope technique. (Broeren & Klein 2023)

At right is an application of the NEWTSS method to identify the optimal subset of spacecraft to be used for the same configuration.

With this subset selection, we are apply the WT method to spatial scales spanning several orders of magnitude. (The impacts of error in timing and spatial knowledge are considered in Klein et al 2024)

